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Sequential Bayesian Optimization Coupled with Differential Evolution for Geological Well Testing

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SUMMARY

This paper presents a novel approach for updating the reservoir model from well test data. A sequential Bayesian optimization technique, i.e. Gaussian Process, is coupled with the Differential Evolution (DE) algorithm, for guided sampling from the parameter space. The Gaussian process assumes the simulation outputs are normally distributed, and aims at modelling the current model and misfit data to predict the best next sampling locations. The next samples are chosen by maximizing the expected improvement gained by sampling from a new location. Differential evolution is used in the maximization process of the expected improvement. This procedure is successfully tested in matching a noisy well test data from a multi-layered faulted reservoir model. The samples from multiple well-test simulations are pooled together, and the Markov chain Monte Carlo (McMC) techniques are used to estimate the posterior distributions over the parameter space. The computational cost of McMC process is reduced by implementing a bootstrap Multivariate Adaptive Regression Spline.

Introduction

Standard well test interpretation techniques entail matching simplified analytical models (from rather complex mathematical solutions) to the pressure transients caused by the rate changes. Although, the analytical models are very useful in many reservoir engineering applications, the inability of the models to fully describe the nonlinearities and reservoir heterogeneities can limit the usefulness of well test data in the process of reservoir characterization.

Geological well testing (Hamdi et al., 2013b) from the detailed 3D static models can largely improve the application of well test data for practical purposes. In particular, the geological well testing will provide a framework to assist in updating the static reservoir models from the information of simulated and observed well test data. Therefore, the scope of well-testing has been enhanced from mainly parameter estimation techniques using analytical models to more sophisticated disciplines for reservoir description and updating (Du and Stewart, 1992). The updating process of the static model from numerical transient test simulations involves three distinct steps; parameterization, forward simulation and inversion. The inversion process involves minimization of an objective function, which in our case is the residual sum of squares of drawdown derivatives in the logarithmic domain.

For such cases, we are not promoted to use the gradient based methods (Gilman and Ozgen, 2013) for optimization as they heavily depend on the starting point and cannot guarantee to find the global minima in high dimensional problems (Hamdi et al., 2013a). In this work, we implement the differential evolution algorithm coupled with a Bayesian Optimization (BO) technique (Lizotte, 2008), i.e. Gaussian Process (GP). The GP (Jones, 2001) algorithm has proven to be efficient in a wide range of engineering optimization problems using a limited number of simulations (Azimi et al., 2010).

We present a realistic well test response of a multi-layered faulted reservoir. We use the Gaussian Process to model the well test misfit in each iteration, and use the Maximum Expected Improvement (Jones, 2001) as a selection criterion for designing the best parameters for the next simulation run. The generated samples (simulation models) during the optimization process are then used within a Bayesian framework to address the uncertainty and predict the posterior distribution over the parameter space using Markov chain Monte Carlo (McMC). To reduce the computational cost during McMC process, we use the Multi-Adaptive Regression Spline (MARS) with bootstrapping (Friedman, 1991) to add more samples at the boundaries of parameters. The statistical analysis and McMC runs are performed using PSUADE package (Tong, 2013).

Gaussian Process (GP)

A Gaussian Process (GP) is probabilistic function, which can model the unknown function at a new location x_i by assuming a output as a Gaussian random variable y(x_i) with a mean $\mu=k(x_i, \boldsymbol{x_o})k(\boldsymbol{x_o},\boldsymbol{x_o})^{-1}\boldsymbol{y_o}$ and a variance $\sigma^2 = k(x_i, x_i)- k(x_i, \boldsymbol{x_o})\, k(\boldsymbol{x_o}, \boldsymbol{x_o})^{-1}\, k(\boldsymbol{x_o}, x_i)$, where ($\mathbf{x_o}$, $\mathbf{y_o}$) is a set of observed input and output data, and $k(.)$ is an arbitrary kernel (Azimi et al., 2012). If we assume a Gaussian kernel, i.e. $k(x_j, x_i)$=exp($||x_i-x_j||/\lambda$), to describe the observed data relationships, the optimal values of mean (μ) and variance (σ^2) are obtained from maximization of exponential likelihood function (λ is the characteristic length scale). When the optimal mean and variance are known, the value of function at a new sample location (x_i) is estimated by maximizing the likelihood function, after adding this new point to the previous observed data. It can be readily shown that (Jones, 2001) for such conditions the optimum function value, $y(x_i)$, which can maximize the likelihood function, is the simple kriging predictor with a variance of $S_D^2(x_i)$.

Maximum expected Improvement (MEI)

The expected improvement, E(I), measures the improvement we can expect by sampling from a new location. If $y(x_i)=y_{max}-I$, where I is the improvement and y_{max} the current best value of function, then the expected improvement is defined as follows (Jones, 2001)

$$\mathrm{E}(I)=\int_{0}^{I=\infty} I\times\left[\frac{1}{2\pi S_D(x_i)}\exp\left(-\frac{y_{\max}-I-y(x_i)^2}{2S_D^2(x_i)}\right)\mathrm{d}I\right]=S_D(x_i)\left[zP(z)+\rho(z)\right]$$

In which, $z=\frac{y_{\max}-y(x_i)}{S_D(x_i)}$, $P(u)$ and $\rho(u)$ the Gaussian cumulative distribution and density functions, respectively. The maximum expected improvement seeks to maximize the improvement we get if we sample from a new location x_i. For this purpose, we use the differential evolution (DE) algorithm (Storn and Price, 1995) which serves as an efficient population-based global optimization technique (Hajizadeh et al., 2010). DE starts with an initial random population of the parameter space and evaluates the corresponding function values for each member in this population. In the DE/random variant of this algorithm, the population members are randomly selected and mixed together to generate the next set of solutions. Hamdi et al. (2014) presented the application of DE to the geological well testing with promising results.

After this step, we run the reservoir simulation to evaluate the well test derivative misfit value for the selected location (i.e. model parameters) by MEI and DE. Then we iterate through the next locations until we touch the minimum misfit threshold, or have reached the maximum number of simulation runs assigned to the algorithm. The overall convergence rate of the GP is measured by the simple regret value that is defined as y_{opt}-y_{max}, where y_{opt} is the optimal value of function and y_{max} is the current best value of the evaluated function.

It should be noted that the accuracy of the algorithm depends on the ability of the GP to adequately represent the simulation evaluations (misfit values). Therefore, it is necessary to generate an initial population of models to act as the kick-off observed data ($\mathbf{x_o}$, $\mathbf{y_o}$) and the next samples are then sequentially added to this initial set accordingly.

Case study

We attempt to match a realistic well test response of a complex reservoir using the GP, MEI and DE. The model includes 40×45×28 coarse cells in x-, y- and z- directions respectively. Each cell covers an average area of 60×60 ft2 and the reservoir thickness is around 140 ft. Fig.1 shows the structural framework of the reservoir model. A fully penetrated vertical well is located close to the center of the model near two cross-cutting normal faults (around 200ft from each fault). The wells produce single phase oil at the rate of 980 STBO/D for almost 14 days of drawdown. A single-phase black oil reservoir simulator (e.g. CMG) was used to simulate the drawdown well test responses. Logarithmic time stepping along with an extensive Cartesian local grid refinement (C-LGR) around the wellbore are employed to capture the early time well test phenomena and reduce the associated numerical artifacts on the well test results (Hamdi et al., 2013a).

Table 1 shows the 8-dimensional parameter space (including K_H for each layer, K_V/K_H, and fault transmissibilities), truth model, and the prior ranges of the reservoir parameters, which are used in the geological well test matching process. The results of three GP runs are shown in Fig.2, where each run includes 250 simulation evaluations. The figure shows that a reasonable match was obtained in less than 100 simulations (for the GP-Run 1 a match after 25 simulations was achieved). The convergence behavior is also favorable in these cases. For this exercise, the GP model is set up with a random initial population of 5, λ=15.11 (summation of the parameter range lengths which was found to be a good approximation). For the MEI optimization, the DE/Random was set up with an initial population and of 400 and 500 iterations. The best model corresponding to the best simulation in GP-Run 1 with the lowest misfit is shown in Fig. 3. An excellent match to the noisy realistic drawdown derivative is obtained.

The simulation populations of these four independent GP runs were pooled together for uncertainty analysis. A cross-validated MARS model with 160 basis function and an interaction level of 7 were used to represent the simulation runs. We assumed all parameters are normally distributed over the parameter ranges. This was then used in a Bayesian framework to predict the posteriores over the parameter space. Fig.4 shows the priors and the corresponding posterior probability distributions of the model parameters. We applied the McMC and Gibbs sampler with one million Monte Carlo samples from the calibrated MARS model. Whenever the posterior doesn't change (e.g. for X6 and X7), no or little information was imposed by the well test data to constrain the model according to the collected samples.

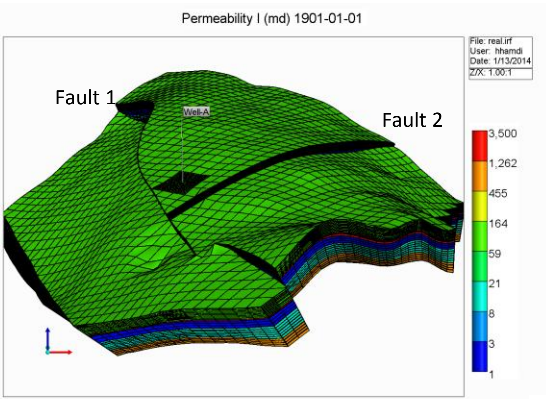

Fig.1: *The multi-layered faulted reservoir model used in this study.*

Parameters	Ranges	Truth case	Best case
X1: $\log(K_H)$ (Layer 1)	0.0 to 3.0	2	1.1
X2: $\log(K_H)$ (Layer 2)	2.7 to 4.0	3.54	3.53
X3: $\log(K_H)$ (Layer 3)	-1.3 to1.7	0.0	-0.42
X4: $\log(K_H)$ (Layer 4)	-1 to 2.3	1.0	1.86
X5: $\log(K_H)$ (Layer 5)	1.0 to 3.03	2.7	2.76
X6: K_V/K_H	0.0 to 1.0	1	0.22
X7: Fault Trans. 1	0.0 to1.0	1.0	0.47
X8: Fault Trans. 2	0.0 to1.0	1.0	0.87

Table 1. *The truth model, parameter ranges and the best model parameters which optioned from GP-Run 1.*

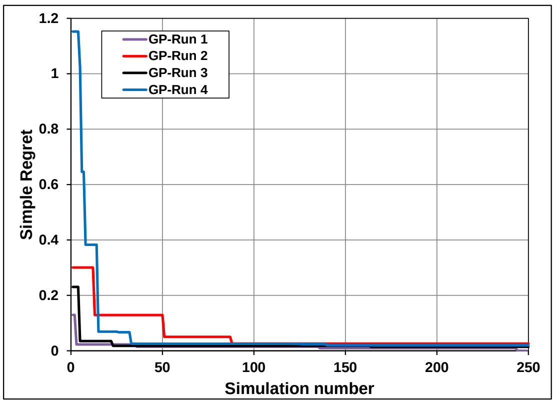

Fig.2: *The simple regret for different GP runs. Ecah GP includes 250 simulations.*

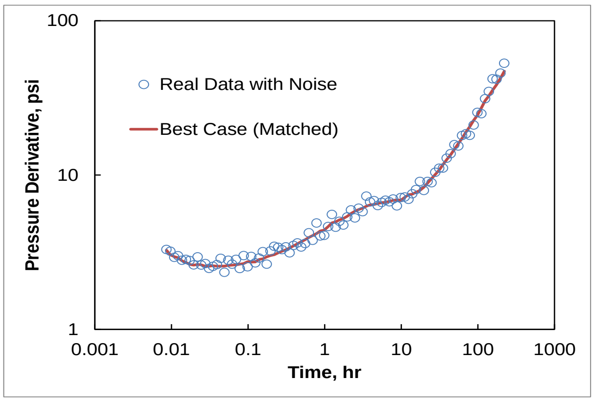

Fig.3: *The well test derivativesf the real case and the best case from GP-Run 1.*

Conclusion

We studied the geological well testing of a faulted multi-layered reservoir model with 8 unknown parameters. A sequential Bayesian Optimization technique (i.e. GP with MEI) coupled with the differential evolution algorithm was successfully implemented in a reservoir engineering problem. The results are promising; with a low number of simulations to achieve a well test derivative match. The results were used in an McMC process, and the posteriors were deducted. For some parameters, the posterior did not change, which was indicative of insensitivity with respect to those parameters. However, the posterior was updated for the other parameters, which clearly indicates the value of well testing in updating the reservoir models.

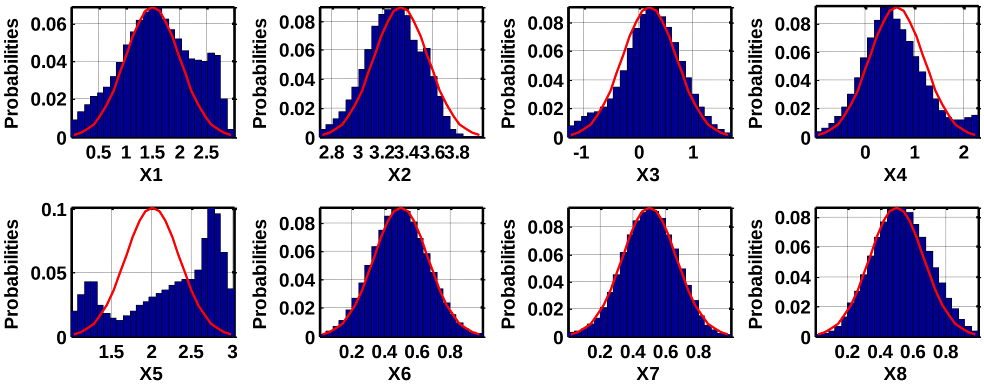

Fig.4: *The assigned Gaussian prior distribution (the Gaussian red curves), and the corresponding estimated posterior histograms for all parameters which are obtained from the Bayesian inference using McMC method.*

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