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# A Soft and Law-Abiding Framework for History Matching and Optimization under Uncertainty

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# Abstract

Current frameworks for optimization and assisted history matching lack the ability to control and guide the sampling engine and to incorporate geo-engineering knowledge. Defining the interactions between uncertain parameters and handling multiple constraints are also arduous tasks. Despite recent advances in adaptive population-based sampling algorithms and other gradient and ensemble-based methods, these specific drawbacks have left engineers with several history-matched models that are inconsistent with the physical and geological knowledge of the field.

We introduce a novel rule-based framework based on fuzzy reasoning to integrate engineering knowledge with optimization and assisted history matching workflows. The system can handle multiple complex constraints both in parameter and objective function space. The use of fuzzy set theory in this workflow is a natural way to address uncertainty arising from imprecision of definition. This type of uncertainty is important in expressing the parameters of interest; however, it has been less addressed in existing workflows. The proposed system can be coupled with any algorithm used for assisted history matching, including gradient-based, population-based and particle filter approaches.

The framework is coupled with differential evolution algorithm and is tested for three cases. The results show that fuzzy rulebased engine preserves the computational efficiency of the sampling engine, while allowing for definition of flexible rules in history matching and optimization that honor engineering knowledge.

# Introduction

History matching involves calibration of a reservoir simulation model by conditioning it to field dynamic data. The basic idea is centered on a belief that a model is more probable to provide accurate predictions if it can successfully reproduce the historical behavior of the reservoir. Dating back to the early 60s [Kruger, 1960] [Jacquard and Jain, 1965], history matching was done manually by reservoir engineers. The end result, after a time-consuming process, was usually a single history-matched model. However, history matching is an inverse problem with non-unique solutions (i.e. different combinations of input parameters may provide a satisfactory match). To have reliable estimates of future field performance, it is essential to have multiple history-matched models with a diverse range of production behaviors to represent uncertainty.

It was in the early 90s that the power of stochastic sampling algorithms came to help engineers in history matching and field optimization. Pioneering works were presented by Ouenes et al. [1992] using simulated annealing (SA) and Sen et al. [1995] using genetic algorithm (GA). The journey continued with evolutionary strategies [Schulze-Riegert et al. 2001], Neighbourhood Algorithm (NA) [Christie et al. 2002] and scatter search [Sousa et al. 2006]. Several factors make these algorithms an excellent choice for tackling history matching problems. As they work with a population of solutions in each generation, an ensemble of history-matched models is produced which can be used to quantify the uncertainty of predictions. These algorithms provide a mechanism to control the exploration/exploitation trade-off through a set of tuning parameters that can be utilized depending on the characteristics of the problem in hand. Population-based algorithms are also easy to implement/parallelize and are more robust in comparison with point-based methods in dealing with noisy objective functions [Nissen and Propach, 1998].

To address the limitations in time and computing power in assisted history matching, recent research has focused on improving the speed and solution diversity of population-based sampling algorithms. Sampling methods used in history matching must be fast in exploring high-dimensional parameter spaces and efficient in finding multiple models using a limited number of simulations. Ant Colony Optimization (ACO) [Hajizadeh, 2010], Differential Evolution (DE) [Hajizadeh et al. 2010] [Rahmati et al. 2011], Particle Swarm Optimization (PSO) [Kathrada, 2009] [Mohamed et al. 2009] and Estimation of Distribution Algorithms (EDAs) [Petrovska and Carter, 2006] [Abdollahzadeh et al. 2011] are all the fruits of research in this direction.

Although population-based methods have a proven track record of success in both academic research and real-world history matching problems, they are often criticized for being a black-box tool. The lack of ability to control and guide the optimization (sampling) algorithm and the difficulty to handle engineering constraints have resulted in many models that perfectly match field data but fail to represent relationships between parameters or satisfy constraints.

In current frameworks used for history matching, usually two types of constraints exist: hard and soft. Hard constraints are not to be violated under any circumstance. If a parameter in the input or/and output space violates a hard constraint, the simulation will terminate and that function evaluation either will be ignored or will be repeated with a new set of parameters. On the other hand, soft constraints are less vital and are allowed to be violated. Should this happen, the sampling effort will continue but a predefined penalty is added to the actual objective function. The goal is to make the violating solution less attractive to the sampling engine, so it is not selected to proceed to the next iteration. Soft constraints can also be used to handle the relationship between input and output parameters.

Adding a penalty in constraint-handling systems, however, may act as a double-edged blade. The problem comes from the fact that it is necessary to define a real number as the threshold of constraint violation. If the parameter value is higher or lower than this threshold, a penalty will be applied to the objective function. Imagine we are defining violation thresholds for a history matching problem and working with porosity and permeability values in several layers of a reservoir. Based on core studies, we have noticed there is relationship between porosity and permeability in the first layer such that if porosity is higher than 15%, then permeability is higher than 130 mD. Also the value of permeability in layer 1 is correlated with permeability in layer 4 due to the geological structure. This creates a complex case for history matching from two distinct aspects. First, the sampling algorithms should be able to handle these complex relationships between parameters and second, the values are based on core sample studies which are uncertain [Chappell and Lancaster, 2007] and may even depend on the lab performing the measurements [McPhee and Arthur, 1994]. A similar scenario can be considered for a field optimization problem where we have concerns about water-cut threshold. In defining the threshold and penalty values, the following critical questions should be asked:

- 1- What is the difference between porosities of 19.999% and 20.001%? Although the difference is negligible from an engineering point of view, this can make an impact on the performance of the sampling algorithm and the penalties applied to objective function.
- 2- How should the uncertainty in the definition of input/output parameters be addressed?

In this work, we introduce a process to incorporate engineering knowledge and handle constraints in assisted history matching and optimization frameworks. This is achieved by integration of a rule-based fuzzy system with population-based sampling algorithms used for history matching and field optimization. The fuzzy inference engine reflects how engineers and software users make decisions regarding the definition of parameter values/relationships and constraint-violation thresholds. The workflow will also make it easier to handle the uncertainty arising from imprecision of definition in input/output search spaces.

#### Fuzzy Sets

Fuzzy set theory was introduced by Zadeh [1965]. He stated that "As complexity rises, precise statements lose meaning and meaningful statements lose precision". Zadeh's theory aims to obtain approximate solutions in problems where we deal with uncertain and vague system descriptions. Fuzzy concepts have blurred boundaries between numbers through computation with words. Unlike Aristotelian logic which looks at the world in a black and white manner, fuzzy logic considers transition zones between numbers or grey colors.

Fuzzy logic can be used to deal with uncertainty. According to Blockley and Godfrey [2000], there are three types of uncertainty that we need to recognize: randomness, fuzziness and incompleteness. Randomness uncertainty is defined as the lack of specific patterns in variables. Incompleteness uncertainty is related to lack of data and refers to what we do not know about the system under study. Incompleteness is the most common source of uncertainty in petroleum engineering and has

been the primary objective for proposing uncertainty quantification workflows. Fuzziness is defined as the imprecision of definition. The imprecision might be due to *the measurement process* or *the way we decide to express the parameters of interest*. This type of uncertainty has not been adequately addressed in history matching problems and it can be treated with fuzzy logic and fuzzy set theory. Fuzzy variables and parameter estimation have been shown to be helpful in uncertainty quantification [Moller et al. 2002] and there are examples where fuzzy logic has been used to handle uncertainty in geoscience applications [Nikravesh et al. 2003] [Wong et al. 2002].

Looking at the reservoir engineering field in general and history matching in particular, where we deal with uncertain description of the underground reservoir, fuzzy logic is an effective way to handle the challenges associated with vague parameter descriptions. Figure 1 shows a fuzzy description of a rock sample and its comparison with an exact (but uncertain) definition.



Porosity of sample is 38.42 % Permeability of sample is 16.21 mili darcy Oil saturation is 31.75%



We have a rock sample with high porosity, low permeability and medium oil saturation

Figure 1: Exact vs. fuzzy description of a reservoir rock sample [Hajizadeh, 2011]

Figure 2 illustrates the smooth transition between sets in fuzzy logic for the permeability example. Fuzzy set *A* is defined by a real value function  $\mu_A(x) = [0,1]$  called the membership function of *A*, which assigns to every element of *x* a real number between 0 and 1 (degree of membership). For example, a permeability value of 44 mD belongs to the set of "low" with a membership degree of 0.18 and to the set of "medium" with a membership degree of 0.82 or:

 $\mu_{\text{Low}}$  (44 mD) = 0.18 and  $\mu_{\text{Medium}}$  (44 mD) = 0.82



Figure 2: Fuzzy membership functions for permeability [Hajizadeh, 2011]

# Approximate Reasoning

Fuzzy reasoning allows decision making based on fuzzy linguistic variables (high, low) and fuzzy operators (and/or). Approximate reasoning is based on fuzzy propositions of the various types in the form of if-then rules. To illustrate the concept, we return to our previous example to determine the oil production using following rules:

IF permeability is low AND porosity is medium THEN oil production is low.

IF permeability is high AND porosity is high THEN oil production is high.

IF permeability is medium AND porosity is medium THEN oil production is medium.

In the set of rules defined for the above example, the terms "low", "medium" and "high" can be defined for porosity and oil production using the same procedure we have used for permeability. The membership functions can take any shape; some

popular choices include triangular, trapezoidal or smooth functions such as Gaussian distributions [Zhao and Bose, 2002]. Figure 3 shows the general framework for a fuzzy inference system. The reservoir engineer's knowledge and historical field data are used to build the fuzzy rule-based system. Crisp inputs are fuzzified using input membership functions and are then processed in the fuzzy inference system. The inference system drives its judgment based on the rule-based section of the workflow and produces fuzzy outputs. These outputs are then defuzzified to obtain crisp values for the output parameters.



Figure 3: Fuzzy inference system for reservoir engineering

#### **Fuzzy Rule-Based History Matching and Optimization**

Most of the methods used for assisted history matching need some higher-level user supervision. The results must be checked to select appropriate outputs and eliminate unrealistic models. A fuzzy rule-based system tends to bring an "intelligent" supervision component to current assisted history matching workflows to eliminate or reduce the need for higher-level human control. This can gradually transform current assisted history matching frameworks to fully automatic frameworks with the expert knowledge being embedded in the system [Iqbal and Dar, 2009].

The essence of this system is a set of IF-THEN rules derived from reservoir engineering knowledge. As shown in Figure 4, a fuzzy rule-based system can check the results of optimization against expert knowledge and previous field data and provide a guideline to the sampling algorithm for producing the next set of solutions.



Figure 4: Suggested workflow for embedding reservoir engineering knowledge in history matching by integrating a fuzzy rule-based system [Hajizadeh, 2011]

#### Integration of Fuzzy Rule-Based System with Differential Evolution

Differential Evolution (DE) is a fairly recent global optimization algorithm for solving problems in continuous space [Storn and Price, 1995]. DE has been proven to be very efficient in different computer science and engineering fields [Price et al. 2005] [Chakraborty, 2008]. Different studies also show that the differential evolution algorithm achieves better results in comparison with other optimization methods such as simulated annealing [Storn and Price, 1995] and different

implementations of genetic algorithms [Cruz et al. 2003] [Biesbroek, 2006].

Like other stochastic methods, DE starts with a randomly generated first generation which consists of  $N_p$  vectors. After initialization of the algorithm and obtaining objective function values, in the second step two vectors are randomly selected among the current population. Then the difference vector between two selected members is computed. In the next step, the difference vector is multiplied by a real number called the scaling factor ( $F \in [0, 2]$ ). The scaling factor controls the amount of perturbation introduced to the difference vector. We then select another vector in the population and we add the scaled difference vector to this new individual. After a crossover stage which increases population diversity, objective function values are evaluated for each vector in the population. Finally, in the selection step, each trial vector competes against the population vector of same index. If the trial vector has a lower objective function compared with the initial individual number, it will be selected for the next generation.

In integrating the fuzzy rule-based system with differential evolution, we follow the original steps for DE and do not modify the internal mechanism. Instead, a penalty function is defined for penalizing solutions that disobey the rule-based system. In other words, DE builds a new set of solutions in each generation and runs a function evaluation (a reservoir simulation) for each member of the population. After function evaluations, the set of input parameters or objective function values are compared with the rule-based system and depending on the degree of disobeyance, a penalty value is added to the original objective function value. In some cases, we define a negative penalty function to actually reward the solutions that fit well with the rule-based system to make them attractive candidates for proceeding to the next generation. The following pseudocodes describe the general workflow.

Fuzzy Rule-Based Differential Evolution	Check C[i] with fuzzy rule-based system
Initialize and evaluate population P	Define the rules for fuzzy system
while (stopping criteria not met) {	
<b>for</b> $(i = 0; i < N_p; i++)$ {	Fuzzify input parameters of $C[i]$
Create candidate $C[i]$	
Evaluate $C[i]$	Pass fuzzy inputs to fuzzy inference engine
Check $C[i]$ with fuzzy rule-based system	
Add penalty	Determine fuzzy penalty value using the rule-
<b>if</b> ( $C[i]$ is better than $P[i]$	based system
P'[i] = C[i]	
else	Obtain fuzzy penalty value
P'[i] = P[i]	
}	Defuzzify the penalty value and add to actual
P = P'	objective function value
}	

#### Examples

We illustrate the integration of fuzzy inference with differential evolution algorithm for three examples. The first example is a benchmark test function for multiobjective optimization. The other examples are based on field and relative permeability history matching problems. Table 1 summarizes the examples, their nature and the specific goals of each problem.

Table 1: Examples used in this paper to illustrate the concept of rule-based optimization and history matching

Example	Nature	Goal
ZDT function	Objective function space	Interaction between multiple objectives
Teal South	Parameter search space	Direct modification of rock and aquifer parameters
SPE-9	Parameter search space	Modification of relative permeability curves

#### ZDT Test Function for Multiobjective Optimization

Multiobjective optimization problems deal with multiple criteria (often in competition or in contrast with each other) that should be minimized or maximized. In these problems, we are looking to find the true Pareto front which indicates the optimal solutions for the multiobjective problem. This means the solutions on the Pareto front cannot be improved in one objective without causing some degradation to the other objective(s). A good multiobjective optimization algorithm should converge to the Pareto front and at the same time maintain diversity of the solutions on the front. Some common examples of multiobjective problems in petroleum engineering include maximizing oil recovery while minimizing the injected gas in gas lift [Ray and Sarker, 2006] or maximizing oil production while minimizing treatment costs in designing hydraulic fracturing operations [Rahman et al. 2001].

Zitzler [et al. 2000] proposed a standard benchmark test suite with six functions for multiobjective optimization problems. In this paper, we have selected the ZDT3 function to test our new workflow. The test function has 30 decision variables and discontinuities in the Pareto-optimal front. It is characterized by the following equations:

$$f_1(x) = x_1$$
  

$$f_2(x,g) = g(x) \cdot \left(1 - \sqrt{\frac{f_1}{g(x)}} - \frac{f_1}{g(x)} \cdot \sin(10\pi f_1)\right)$$
  

$$g(x) = 1 + \frac{9}{n-1} \cdot \sum_{i=2}^n x_i$$

where n = 30, and  $x_i$  [0,1]. The Pareto front is formed with g(x) = 1 and is displayed in Figure 6 with red lines. Previously it was shown that a large number (25000) of random solutions was not able to capture the Pareto front and a multiobjective differential evolution algorithm using Pareto ranking was proposed to tackle this problem [Hajizadeh et al. 2011]. In this work, we aim to solve the problem using a single global-objective differential evolution algorithm coupled with the fuzzy rule-based system. In this example, a global objective function is formed by summing up  $f_1$  and  $f_2$  values. Differential evolution is then used to minimize this global objective function. We use the "best" strategy in DE which takes the vector with the best (lowest) objective function value in the previous generation and uses it as the base vector for building the next generation solutions. Table 2 summarizes the tuning parameters used for this test.

Table 2: Tuning parameters of differential evolution algorithm with "best" strategy in ZDT3 function

$N_p$	F	$C_r$	Generations
100	0.3	0.3	250

For the ZDT3 function, we have designed the fuzzy rule-based system in the objective function space. Figure 6(a) shows the performance of a single-objective DE algorithm for finding the Pareto fronts indicated by red lines. All solutions on the Pareto front are considered to be optimum; however, the algorithm can only identify the front in the lower right corner (higher values for objective 1 and lower values for objective 2). To tackle this problem, a fuzzy rule-based system is designed to favor the solutions which lead to lower values for objective 1 and higher for objective 2 with a hope to cover all possible Pareto fronts. Figure 5 shows the rule-based system with two axes for objectives 1 & 2 and a third axis for the penalty value. If the solutions of DE provide a higher value for objective 1, they will be heavily penalized. Both inputs (objective 1 and 2) and output (penalty value) are defined in the range of (0,1). This range is divided into five regions using Gaussian membership functions. These membership functions will be used to fuzzify and defuzzify input/output parameters in the fuzzy inference system. Table 3 summarizes the mean and variance of the Gaussian fuzzy membership functions used to describe the parameters and penalty function value.

Table 3: Mean and variance for Gaussian membership functions used for input and output parameters in ZDT3 function

	lowlow	low	mid	high	highhigh
Inputs (Obj1, Obj2), Output (Penalty)	0, 0.2	0.25, 0.2	0.5, 0.2	0.75, 0.2	1, 0.2

The following rules are defined for the fuzzy inference system:

- If (objone is highligh) and (objtwo is lowlow) then (penalty is highligh)
- If (objone is highligh) and (objtwo is low) then (penalty is high)
- If (objone is low) and (objtwo is highligh) then (penalty is low)

Figure 6(b) shows that using a single-objective DE algorithm coupled with fuzzy inference, we are able to identify more disconnected Pareto fronts. The results indicate that a fuzzy penalty function can help to guide the differential evolution algorithm towards the regions of interest in objective function space (higher values for objective 2, lower value for objective 1).



Figure 5: Fuzzy system for determining penalty value for the objective function in ZDT 3 example



Figure 6: (a) Original DE result for ZDT-3 function and (b) DE integrated with fuzzy rule-based system

#### **Teal South Reservoir**

The second example we use to demonstrate the application of our rule-based system is a history matching problem. The Teal South reservoir is located in block 354 of Eugene Island in the Gulf of Mexico. The reservoir simulation model is set up on a  $11 \times 11 \times 5$  corner point grid (Figure 7-left) [Hajizadeh, 2011]. The reservoir history data consist of monthly production rates of oil, gas and water for 3.5 years (Figure 7–right). There are five geological layers in the model with uniform properties. The unknown parameters in history matching are horizontal permeability multipliers for each of these five layers (P1-P5), a single value for vertical to horizontal permeability ratio (P6), rock compressibility (P7) and aquifer strength (P8). History matching is only done on field oil production rate using all available production data. Parameterization for the Teal South model and their prior ranges are shown in Table 4.

Parameters	Units	Prior range
k <sub>h</sub> (for each layer)	mD	10 - 1000
k <sub>v</sub> /k <sub>h</sub>	-	$10^{-4} - 10^{-1}$
Rock compressibility	psi <sup>-1</sup>	5×10 <sup>-6</sup> - 1×10 <sup>-4</sup>
Aquifer strength	MMSTB	$10^7 - 10^9$

Table 4: Parameterization and prior ranges for the Teal South model



Figure 7: Teal South reservoir model (left) and production history of the reservoir (right)

The following objective function is used to calculate the misfit value for the Teal South model:

$$M = \sum_{n=1}^{N} \frac{(q_{obs}(t_i) - q_{sim}(t_i))^2}{2\sigma^2}$$
(1)

where N is the number of observations, q is the flow rate for observed and simulated oil production data, and  $\sigma^2$  is the variance of the observed data.

Based on core analysis, we assume two scenarios to demonstrate the fuzzy rule-based system. Both scenarios consider a relationship between horizontal permeability in layers 3 and 4 (P3, P4). In the first case we assume that if P3 is low or very low, then P4 should be medium. In the second case, engineers believe that if P3 is medium, then P4 should be medium too. It should be remembered that the "linguistic" rules are backed with fuzzy membership functions. In the Teal South example, the range for P3 and P4 is defined in (0,1) and for the penalty value, the range is in (-5,30). After the parameter values are selected for P3 and P4, they are scaled to the original range stated in Table 5 and are replaced in the simulator's input deck. Table 5 summarizes the mean and variance for the Gaussian membership functions used in the fuzzy inference system for input and output parameters. For example, a "low" value for P3 is characterized by a Gaussian function which has a mean of 0.25 and a variance of 0.2. This allows considering uncertainty in parameter definition and relaxes the engineer in dealing with uncertain numbers and applying his knowledge to history matching.

Table 5: Mean and variance for Gaussian membership functions used for in the Teal South example

	lowlow	low	mid	high	highhigh
Inputs (P3 & P4)	0, 0.2	0.25, 0.2	0.5, 0.2	0.75, 0.2	1, 0.2
Output (Penalty)	-5, 1	3,1	15,1	20,1	30,1

The fuzzy inference system is based on following set of rules for Cases 1 and 2. Figure 8 shows the graphical representations of the fuzzy inference system:

Case 1:

1. If (P3 is low) and (P4 is mid) then (penalty is lowlow)

2. If (P3 is lowlow) and (P4 is mid) then (penalty is lowlow)

3. If (P3 is highligh) and (P4 is lowlow) then (penalty is highligh)

4. If (P3 is lowlow) and (P4 is highligh) then (penalty is high)

5. If (P3 is highligh) and (P4 is highligh) then (penalty is highligh)

6. If (P3 is lowlow) and (P4 is lowlow) then (penalty is highligh)

7. If (P3 is mid) and (P4 is lowlow) then (penalty is high)

8. If (P3 is high) and (P4 is low) then (penalty is highligh)

9. If (P3 is low) and (P4 is highligh) then (penalty is highligh)

Case 2:

1. If (P3 is mid) and (P4 is mid) then (penalty is lowlow)

2. If (P3 is high) and (P4 is mid) then (penalty is low)

3. If (P3 is lowlow) and (P4 is lowlow) then (penalty is highligh)

4. If (P3 is lowlow) and (P4 is highligh) then (penalty is highligh)

5. If (P3 is highligh) and (P4 is highligh) then (penalty is highligh)

6. If (P3 is highligh) and (P4 is lowlow) then (penalty is highligh)

7. If (P3 is highligh) and (P4 is mid) then (penalty is highligh)

8. If (P3 is mid) and (P4 is lowlow) then (penalty is highligh)

9. If (P3 is mid) and (P4 is highligh) then (penalty is highligh)



Figure 8: Rule-based system for determining penalty in the Teal South history matching problem – (a) Case 1 and (b) Case 2

The "best" strategy of DE algorithm with (population size  $(N_p) = 25$ , scaling factor (F) = 0.5, crossover rate  $(C_r) = 0.9$  and 1000 simulations) has been used in history matching of the Teal South reservoir. In addition to Cases 1 and 2, a third history matching run was performed using the same tuning parameters, but without the additional penalty coming from the fuzzy inference system. The best misfit values (Equation 1) for "Case 1", "Case 2" and "no penalty" are 15.93, 16.81 and 16.79 respectively. Figure 9 compares the best match results of these cases with historical oil production rates. As we can see in this figure, all three cases provide satisfactory and almost identical matches to data. Figure 10 shows the boxplots for generational misfit values in these three cases. The boxplots provide a graphical way to analyze the performance of the DE algorithm in each generation and the overall convergence behavior. For each generation, we have plotted min, median and maximum values of the misfit boxplots in Figure 10. The figure demonstrates that the coupling of the fuzzy inference system does not have an adverse effect on the convergence rate of DE algorithm. In the boxplot min section, Case 1 has a lower generational min in comparison with Case 2 and the "no penalty" run. The generational medians are tied, while generational max values are higher for Case 2. The fact that Case 1 has a lower minimum misfit value in each generation for this history matching study may not be conclusive to prove that the fuzzy inference system improves convergence rate in all cases. However in complex real-life history matching studies where extensive knowledge of experts are represented through fuzzy rules, the system can help to improve the computational efficiency of the sampling engine by avoiding exploration in unrealistic regions.

To further study the effect of the fuzzy rule-based system on the performance of DE, we also looked at the sampling history of the algorithm. The sampling history for each case is shown in Figure 11 (a,b,c), which plots the scaled value of the 8 unknown parameters versus simulation number. This figure provides an insight into the performance of differential evolution in sampling the 8-dimensional space and the final regions where the algorithm has rested for each parameter.

**Best History Match** 



Figure 9: Best history matching results in the Teal South example



Figure 10: Comparison of minimum, median and maximum misfit boxplot values in each generation of DE algorithm for the Teal South problem



Figure 11: Sampling trail for Case 1 in (a) and for Case 2 in (b)



Figure 11(c): Sampling trail for unknown parameters using DE algorithm with no penalty (Case 3)

As shown in Figure 11, the fuzzy inference system has successfully guided the DE algorithm towards the regions of interest in the search space. In Case 1, we assumed that if P3 is low or very low then P4 is medium and that is precisely what we see in the sampling history (Figure 11 - a). The inference system has also steered the algorithm to a region where P3 and P4 are both medium in Case 2 (Figure 11 - b). The third case (no penalty was applied) has converged to a different region which violates the engineering assumptions for P3 and P4 (Figure 11 - c). The Teal South example shows that the fuzzy rule-based system can incorporate engineering knowledge and preference into history matching while preserving the computational efficiency of the sampling algorithm.

#### SPE9: History Matching of Relative Permeability Curves

The Ninth SPE Comparative Solution project [Killough, 1995] was designed to study the performance of different black-oil reservoir simulators. The reservoir (Figure 12) is represented by a 24×25×15 mesh with rectangular coordinates. The dimensions of the grid blocks are 300 feet in both the X- and Y- directions. Values of porosity and thickness can be found in the paper by Killough. The total thickness from layers 1 to 13 is 209 feet and layers 14 and 15 have thickness of 50 and 100 feet respectively. Porosity values are reported in the original paper. PVT properties of the model come from the Second Comparative Solution Project [Weinstein et al. 1986].



Figure 12: Grid top view for the 9<sup>th</sup> SPE Comparative Solution reservoir model

In the third example, we history match the relative permeability curves in the SPE9 model. A power-law model [Lee et al. 1987] is used to obtain the curves. Table 6 shows the unknown parameters and their prior ranges. In the fuzzy inference system, we work with the Corey exponent for water and oil. It is assumed that if "nw" is in the higher end of the initial range, then "no" will also be in the same position. The truth values for "nw" and "no" are assumed to be 2.93 and 4.88 respectively. The rules for the fuzzy inference system (Figure 13 - a) are designed to reward the solutions with higher "nw" and "no".

- 1. If (*nw* is highligh) and (*no* is highligh) then (penalty is lowlow)
- 2. If (*nw* is high) and (*no* is high) then (penalty is lowlow)
- 3. If (*nw* is lowlow) and (*no* is lowlow) then (penalty is highligh)
- 4. If (*nw* is lowlow) and (*no* is highligh) then (penalty is highligh)
- 5. If (*nw* is highligh) and (*no* is lowlow) then (penalty is highligh)
- 6. If (*nw* is low) and (*no* is mid) then (penalty is highligh)
- 7. If (*nw* is lowlow) and (*no* is low) then (penalty is highligh)
- 8. If (*nw* is mid) and (*no* is lowlow) then (penalty is high)
- 9. If (*nw* is lowlow) and (*no* is mid) then (penalty is highligh)
- 10. If (*nw* is high) and (*no* is lowlow) then (penalty is highligh)
- 11. If (*nw* is lowlow) and (*no* is high) then (penalty is highligh)

The mean and variance of the Gaussian membership functions for input and penalty values in the fuzzy inference system are presented in Table 7. The nomenclature for the table is presented after conclusions.

Parameter	Range	Parameter	Range	Parameter	Range
swcon	0.05 - 0.25	krwiro	0.2 - 0.7	sgcrit	0.02 - 0.06
iswcrit	0.0001 - 0.2	nw	1 - 3	sgcon	0.0001 - 0.02
soirw	0.05 - 0.25	no	2 - 5	krgcl	0.6 - 1
isorw	0.0001 - 0.2	soirg	0.05 - 0.25	nog	1 - 3
krocw	0.5 - 1	isorg	0.0001 - 0.2	ng	1 - 3

Table 6: Unknown parameters and prior ranges in the SPE9 example

Table 7: Mean and variance for Gaussian membership functions used for in the SPE9 example

	lowlow	low	mid	high	highhigh
nw	1, 0.25	1.5, 0.25	2, 0.25	2.5, 0.25	3, 0.25
no	2, 1	2.5, 1	2.75, 1	4, 1	5, 1
Penalty	-500000, 10000	200000, 50000	1000000, 200000	1500000, 200000	2500000, 200000

Figure 13(b) shows the convergence of DE algorithm with "best" strategy and ( $N_p = 25$ , F = 0.3,  $C_r = 0.5$ , 1000 simulations). Blue cross marks represent the original misfit values and red dots show the objective function values after adding the penalty. The penalized misfit values are used in the DE to evaluate solutions and select candidates for the next generation. For the SPE9 model, the "lowlow" penalty function has a mean value of -500000. If a solution has higher values for "nw" and "no", adding this negative penalty function will actually reduce the original misfit value. Looking at Figure 13(b), we see that in the initial steps of history matching, solutions receive a positive penalty and red dots are mostly placed above blue marks. Progressing towards final solutions, the negative penalty acts as a rewarding mechanism and produces more models with higher "nw" and "no". This is reflected in the convergence graph with more red dots being placed below blue marks. It should be noted that the penalty values must be in harmony with the objective function numbers. Thus it is necessary to have an estimate of the initial misfit values in order to define a proper range for the penalty function. We assumed a truth value of 2.93 for "nw" and 4.88 for "no". The values for "nw" and "no" in the best match for the SPE9 example are 2.79 and 4.82 respectively.



Figure 13: Rule-based fuzzy inference and convergence of DE algorithm for the SPE9 example

Figure 14 shows the ensemble of 1000 models for history matching relative permeability curves, best match and truth curve. Depending on the reservoir rock being water wet, oil wet or intermediate, the fuzzy inference system can be used to generate a more realistic relative permeability curve in real-life history matching problems. Figures 15-17 show the match result for cumulative oil, gas and water production in the SPE9 example after adjusting the relative permeability curves. These figures show a reasonable agreement between simulated and observed values.



Figure 14: Ensemble of relative permeability curves in history matching and the best match vs. truth



Figure 15: Best history matching for cumulative oil production



Figure 16: Best history matching for cumulative gas production



Figure 17: Best history matching for cumulative water production

# Discussion

The rule-based system introduced in this paper to incorporate engineering knowledge in population-based algorithms can also be used to guide other techniques for assisted history matching and optimization. Gradient-based algorithms with penalty functions to satisfy constraints have been sitting on the shelves for the past four decades [Luenberger, 1971] [Luenberger,

1974]. The ordinary penalty function can easily be replaced with the rule-based penalty introduced in this work and be applied to history matching problems. The fuzzy inference system can also be integrated with different flavors of Ensemble Kalman Filter (EnKF). This can be achieved either in the update stage or in hybrid versions of EnKF with evolutionary algorithms [Schulze-Riegert et al. 2009] and MCMC [Emerick and Reynolds, 2012]. One interesting direction to follow is combining the fuzzy rule-based system with the sub-space EnKF [Sarma and Chen, 2012]. Sub-space EnKF uses different parameterization for each ensemble member. The proposed approach can be used to favor ensembles that honor the desired geological/geostatistical pattern and reject the ones that deviate from engineering constraints.

Integration of geological information into history matching and field optimization is another area that can benefit from the introduced framework. In history matching using black-box sampling algorithms, it is important to test different geological interpretation scenarios and study the impact of incorporating a hierarchy of heterogeneities into geological models. For example, distributions of thin, low permeability units with extensive aerial continuity can significantly affect the fluid flow [Lun et al. 2012]. Furthermore, geologists may have prior information on complex geological structures when parameterizing a model for history matching. For example, this information can be in the form of relationships between thickness, width, wavelength and amplitude values used to generate sinuous channels in reservoir models [Rojas et al. 2011]. The experience of geologists can be used to create a rule-based fuzzy inference engine to control the geological structures and eliminate unrealistic models in history matching. The same idea can be used in geostatistical parameterization for history matching. For example a relationship may be defined between different parameters of variogram for each layer. The fuzzy inference system will then penalize the realizations that do not follow the desired patterns and hence reduce the chance for these models from being carried to next iterations. On the other hand, it would be very interesting to understand the effect of fuzzy inference on the uncertainty of predictions after the history matching step.

Fuzzy set theory can also be used to define fuzzy objective functions in history matching. Rommelfanger [2007] provides a critical survey of different methods in optimization of fuzzy objective functions. The application of fuzzy functions for multiobjective optimization of electromagnetic devices has been demonstrated by Chiampi [et al. 1998]. It is also possible to define distance metrics to measure performance of population-based algorithms and combine information from these metrics with a rule-based inference system to guide the sampling engine [Hajizadeh et al. 2012].

In this work, fuzzy membership functions are used to deal with uncertain parameters. These membership functions can be defined in various shapes such as triangular, trapezoidal, Gaussian, etc.; but they all come with fixed end-points (crisp). If there is some uncertainty about the end points of membership functions, Type-2 fuzzy systems can be used [Mendel, 2007]. Unlike crisp fuzzy membership functions in Type-1 systems, Type-2 fuzzy systems have fuzzy membership functions which can be useful in cases where it is difficult to determine exact membership functions for input/output parameters. Some of these ideas will be pursued in upcoming work.

#### Conclusions

We introduced a novel mechanism to incorporate engineering knowledge in assisted history matching through combining a fuzzy-inference system with the sampling engine. Three examples demonstrate the application areas and show that the rulebased system preserves computational efficiency of the original sampling algorithm. We have shown that using this novel approach, we are able to guide the sampling algorithm to regions of interest. The law-abiding system makes it easy to define and handle complex constraints both in parameter and objective function space. The system also elegantly deals with interactions among variables. Finally, the introduced workflow opens a door for integration of reservoir engineering and geological knowledge in gradient and ensemble-based methods used in history matching and production optimization.

#### Nomenclature

swcon:	Connate water saturation
soirw:	Irreducible oil saturation
krocw:	Relative permeability value of oil at the connate water saturation
krwiro:	Relative permeability value of water at the irreducible oil saturation
nw:	Corey exponent for water in water and oil relative permeability system
no:	Corey exponent for oil in water and oil relative permeability system
soirg:	Irreducible oil saturation in the two phase gas liquid system
sgcrit:	Critical gas saturation
sgcon:	Connate gas saturation
krgcl:	Relative permeability value of gas at the connate liquid saturation
nog:	Corey exponent for oil in liquid and gas relative permeability system
ng:	Corey exponent for gas in liquid and gas relative permeability system

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