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# Gaussian Process for Uncertainty Quantification of Reservoir Models

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## Abstract

Reservoir history matching is a computationally expensive process, which requires multiple simulation runs. Therefore, there is a constant quest for more efficient sampling algorithms that can provide an ensemble of equally-good history matched models with a diverse range of predictions using fewer simulations. We introduce a novel stochastic Gaussian Process (GP) for assisted history matching where realizations are considered to be Gaussian random variables. The GP benefits from a small initial population and selects the next best possible samples by maximizing the expected improvement (EI). The maximization of EI function is computationally cheap and is performed by the Differential Evolution (DE) algorithm. The algorithm is successfully applied to a structurally complex faulted reservoir with 12 unknown parameters, 8 production and 4 injection wells. We show that the GP algorithm with EI maximization can significantly reduce the number of required simulations for history matching. The ensemble is then used to estimate the posterior distributions by performing the Markov chain Monte Carlo (McMC) using a cross-validated GP model. The hybrid workflow presents an efficient and computationally-cheap mechanism for history matching and uncertainty quantification of complex reservoir models.


## Introduction

In making business plans for development of oil and gas resources, having a reliable prediction of reservoir performance is of prime importance. The history matching process aims to address this concern where the simulation model is updated to reproduce the historical behavior of the field. The outcome of a history matching exercise is then used to quantify the prediction uncertainty of reservoir models. History matching is an inverse and ill-posed problem with multiple non-unique solutions. Despite the advances that have been made during the past 55 years, the history matching has remained to be a challenging task for reservoir engineers. The improvements in computational algorithms and resources have been masked by moving towards billion-cell simulation models. Therefore, the total amount of time and resources required to run several simulation models during a history matching study has remained relatively unchanged. These challenges have kept the history matching and uncertainty quantification research a very live topic in our community.

Three main categories of algorithms exist in the literature to tackle the history-matching problem. These include gradient-based algorithms (Slater and Durrer (1970), Watson and Lee (1986)), population-based approaches (genetic algorithm (Romero et al., 2000), evolution strategy (Schulze-Riegert et al., 2001), differential evolution (Hajizadeh et al., 2009), estimation of distribution algorithms (Abdollahzadeh et al., 2011)) and particle filter methods (Naevdal et al., 2003). The majority of assisted history matching packages adopted by the industry are powered by stochastic population-based sampling algorithms.

Population-based algorithms are favorable because they produce an ensemble of history-matched models that can be used during the uncertainty quantification stage. Tavasolli et al. (2004) showed that prediction of the future performance of a reservoir based on a single best history matched model may not be reliable. Population-based algorithms also offer more robustness in dealing with noisy objective function in comparison with point-based methods (Nissen and Propach, 1998). However, a requirement to run multiple simulations has remained a major bottleneck for the application of these algorithms for large simulation cases. Furthermore, the efficiency of the sampling algorithm is critical for production optimization problems where the same workflow can be used to maximize an objective function value (Ding, 2008). In this paper, we look at a particular class of population-based history matching algorithms which takes the advantage of the modeling the simulation outputs using a multi-Gaussian distributions. Such an assumption guides the selection of better samples as the history matching iterations advance. We first describe the Gaussian Process (GP) as an efficient surrogate, and then discuss its application for history matching of a medium sized faulted reservoir.

## Gaussian Process

A Gaussian Process (Rasmussen and Williams, 2005) is used to model a desired system output y($x_i$) and assumes that it follows a multi-Gaussian distribution with a mean $\mu = k(x_i, \mathbf{x_o}) k(\mathbf{x_o}, \mathbf{x_o})^{-1} \mathbf{y_o}$ and a variance $\sigma^2 = k(x_i, x_i) - k(x_i, \mathbf{x_o})\, k(\mathbf{x_o}, \mathbf{x_o})^{-1}\, k(\mathbf{x_o}, x_i)$. With such an assumption, we are able to set a mathematical framework to estimate the unknown function at a new location ($x_i$). In this process, ($\mathbf{x_o}$, $\mathbf{y_o}$) is a set of observed input and output data, and $k(.)$ is an arbitrary kernel or covariance function (Azimi et al., 2012). If we assume a Gaussian covariance function – i.e. $k(x_j, x_i) = exp(||x_i - x_j|| / \boldsymbol{\lambda})$ – to describe the observed data relationships, the optimal values of the GP's mean and variance are obtained from maximization of exponential likelihood function ($\boldsymbol{\lambda}$ is the characteristic length scale vector which describes the variability of each input parameter). When the optimal mean and variance are known, the value of function at a new sample location ($x_i$) can be estimated. Jones (2011) showed that for such conditions the optimum function value, $\hat{y}(x_i)$ that maximizes the likelihood function is the ordinary kriging predictor with a variance of $S_D{}^2(x_i)$.

## Maximum expected Improvement (MEI)

The expected improvement, EI is a measurement of the anticipated enhancement one expects by sampling from a new location. Assuming $y(x_i) = y_{max} - I$, where $I$ is the improvement and $y_{max}$ the current best value of function, then the EI is defined as the following (Jones, 2001):

$$\mathrm{E}(I) = \int_{0}^{I=\infty} I \times \left[ \frac{1}{2\pi S_D(x_i)} \exp\left( -\frac{y_{\max} - I - y(x_i)^2}{2S_D^2(x_i)} \right) \mathrm{d}I \right] = S_D(x_i)\left[ zP(z) + \rho(z) \right] \quad (1)$$

In which, $z = \frac{y_{\max} - y(x_i)}{S_D(x_i)}$ , $P(u)$ and $\rho(u)$ the Gaussian cumulative distribution and density functions, respectively. The maximum expected improvement seeks to maximize the improvement we get if we sample from a new location $x_i$. For maximizing the EI, we use the DE/Best variant of the Differential Evolution (DE) algorithm which is an efficient population-based optimization technique (Storn and Price, 1995; Hamdi et al., 2015). Although, DE requires a large number of function evaluations, this would not create any problem for maximizing the EI as this analytical function is very cheap to evaluate. Such a procedure results in proposing a new sample location $x_i$ that can potentially have a lower misfit value. The proposed sample is passed to the reservoir simulator and the actual misfit value is evaluated. Using this procedure, more and more samples are sequentially added until we touch the minimum misfit threshold, or have reached the maximum number of simulation runs assigned to the algorithm. The success rate of the algorithm relies on the quality of the GP model to efficiently emulate the simulator's output (misfit values). As such, it is essential to generate an initial population of models to act as the starting data ($\mathbf{x_o}$, $\mathbf{y_o}$) and the next samples are then sequentially added to this initial set. The overall performance of GP is assessed by introducing the *regret* function rather than the actual misfit value and is defined as $y_{opt}$-$y_{max}$, where $y_{opt}$ is the optimal value of function (i.e. zero) and $y_{max}$ is the current best value of the evaluated function. Figure 1 shows the overall workflow for GP optimization and uncertainty quantification that is introduced in this work.

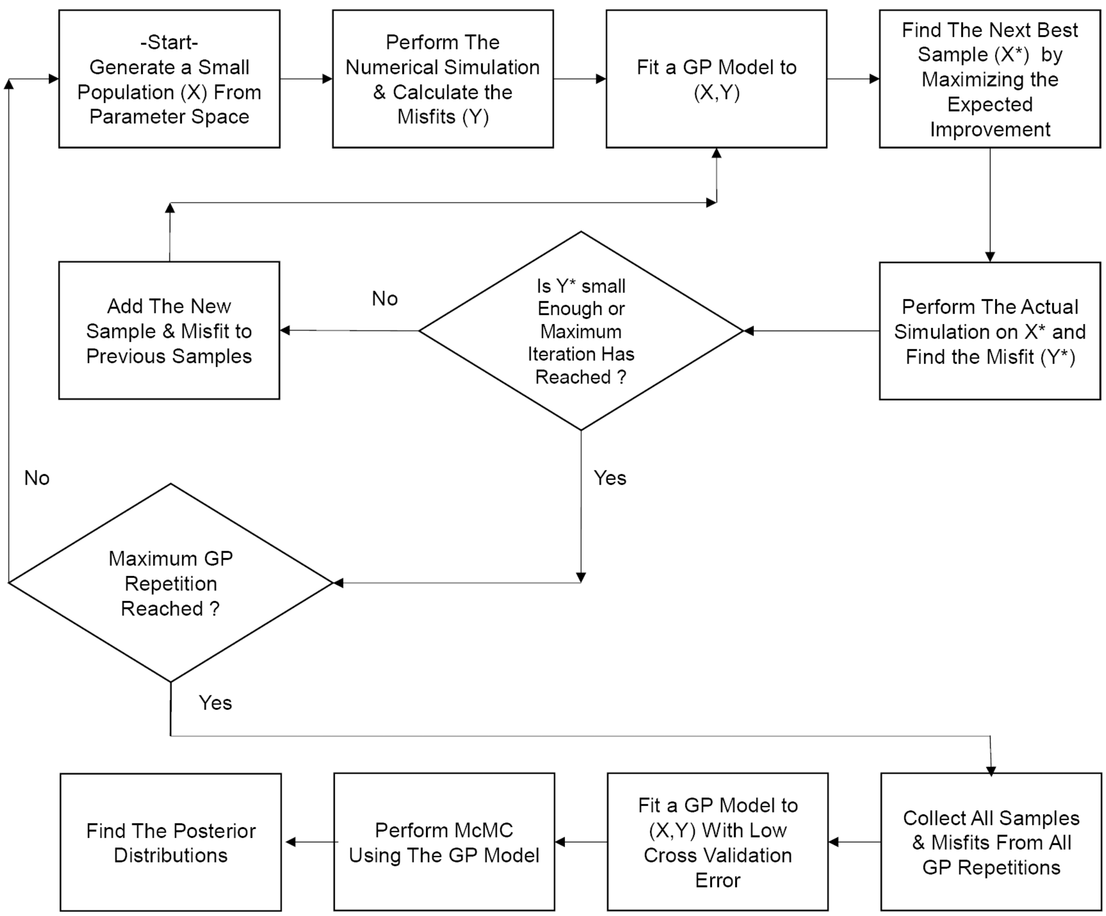


**Figure 1—An uncertainty quantification workflow using the GP modeling that is implemented in this paper**

## Description of the Reservoir Model

The reservoir model used in this study is a faulted fluvial reservoir model. The model consists of 25600 cells (40×40×16). Each cell has a dimension of 350ft×350ft×7.5ft. We consider a two-phase flow of water and oil in the reservoir with no gas being present. CMG-IMEX™ black oil reservoir simulator (Computer Modelling Group, 2014) is used to simulate the fluid flow. For this model, the WOC is located at 6555 ft. with reference depth being 5560 ft. The pressure at the reference depth is 5000 psi and the bubble point pressure is 1000 psi. The model has 3 distinct faults and 12 wells: 4 injectors (I01, I03 to I05) and 8 producers (P01 to P08). The injection wells are controlled on water rate and the producers with the bottomhole pressure. Figure 2 shows a snapshot of the 3D model with the locations of the wells in the reservoir.

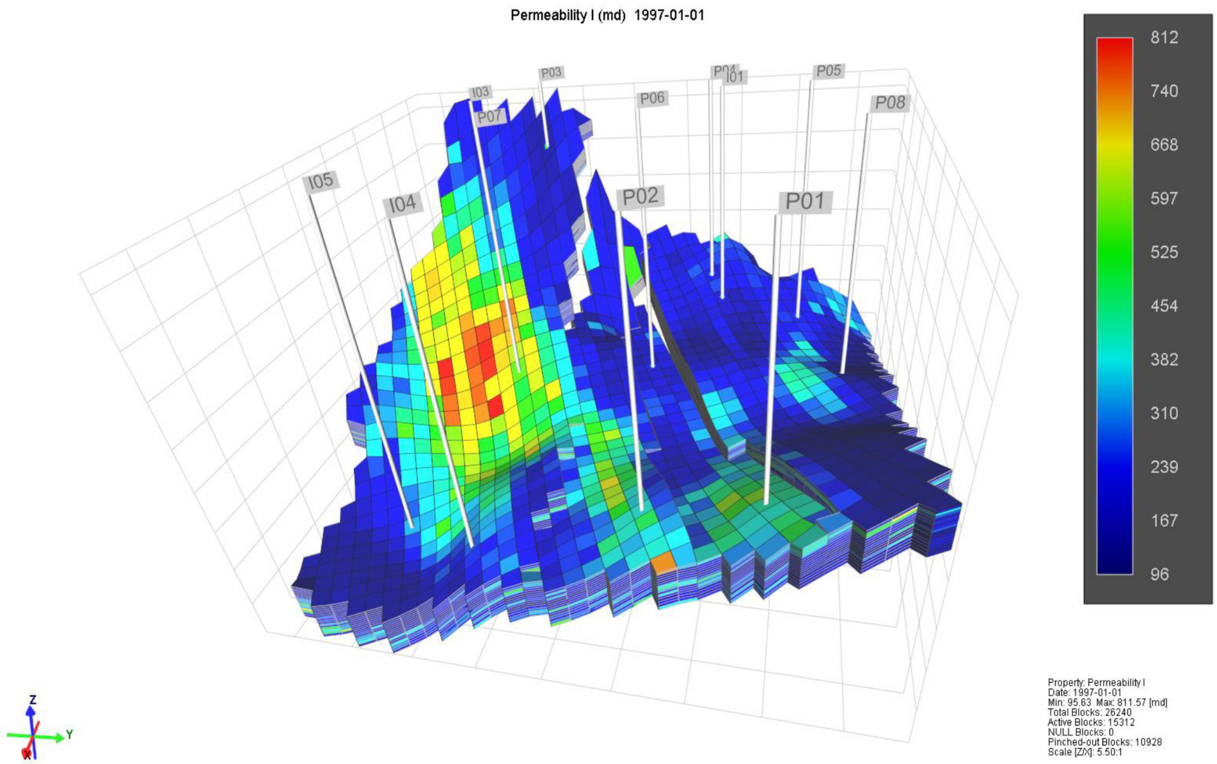


**Figure 2—The reservoir model that is used in this study**

The model parameters and initial ranges and their truth values for the history matching problem are listed in Table 1. The parameters include the multipliers for the anisotropic permeability field and the faults, the aquifer size, end-point relative permeability and the Corey's exponents $N_w$ and $N_o$ (Corey, 1954) for generating the water and oil relative permeability curves. The combination of parameters presented in Table 1 poses a 12-dimensional history matching problem.

**Table 1—History matching parameters, initial ranges and the truth model values**

| Parameter | Min | Max | Truth |
|---|---|---|---|
| $k_x$ Multiplier (X1) | 0.2 | 2 | 1 |
| $k_y$ Multiplier (X2) | 0.2 | 1.5 | 1 |
| $k_z$ Multiplier (X3) | 0 | 1.5 | 1 |
| F2 Multiplier (X4) | 0 | 1 | 0.5 |
| F3 Multiplier (X5) | 0 | 1 | 0 |
| F4 Multiplier (X6) | 0 | 1 | 1 |
| Aquifer Length, ft (X7) | 300 | 30000 | 20000 |
| Aquifer Permeability, md (X8) | 10 | 1000 | 500 |
| $N_w$ Corey's exponent for $K_{rw}$ (X9) | 1.5 | 5 | 2 |
| $N_o$ Corey's exponent for $K_{ro}$ (X10) | 1.5 | 5 | 2 |
| $K_{ro}$ at $S_{wi}$ =0.25 (X11) | 0.6 | 0.95 | 0.9 |
| $K_{rw}$ at $S_{or}$ =0.15 (X12) | 0.1 | 0.6 | 0.5 |

## Results

We use a Latin Hypercube design to generate 10 initial samples from the parameter space and calculate the corresponding misfit values. The scaled misfit ($M^2$) between the observed (*obs*) and simulation (*sim*) data is defined over all wells (W) and timesteps (T) as follows:

$$M^2 = 5 \times \sum_W \sum_{t=1}^{T} \left( \frac{q_{o\_obs} - q_{o\_sim}}{\sigma_o \overline{q_{o\_obs}}} \right)^2 + \sum_W \sum_{t=1}^{T} \left( \frac{WC_{obs} - WC_{sim}}{\sigma_{wc} \overline{WC_{obs}}} \right)^2 + 10 \times \sum_W \sum_{t=1}^{T} \left( \frac{P_{inj\_obs} - P_{inj\_sim}}{\sigma_p \overline{P_{inj\_obs}}} \right)^2 \quad (2)$$

in which, WC is water cut, $q_o$ is the producer's oil rate (STB/day), $\sigma$ is the assumed error in each measurement, and $P_{inj}$ is the injector pressure (psi). The misfit value is scaled with respect to the average values of the observed data and assumes measurement errors of 1% for rates and 20 psia for pressures. The constant coefficients of 5 and 10, which appear in the misfit definition, are used to ensure that all individual misfit values fall within the same numeric ranges. These constants are obtained by running an initial set of independent random samples from the parameter space and comparing the individual misfits before running the GP algorithm. These samples are used to fit a GP model to the data to sequentially propose the next best sample by maximizing the expected improvement. The best GP hyper-parameters (mean, variance and the length scales) are obtained by maximizing the log-likelihood. In this problem, we set a maximum of 350 iterations and will repeat the optimization for 4 times. The convergence is shown based on the average performance of these independent GP runs. The performance of GP is understood from the calculated *regret* values.

Figure 3 shows the convergence of the GP that is obtained by averaging the all regrets over four individual runs. The regret has been reduced logarithmically from around 10000 to around 30 in less than 350 simulations for this problem. This shows the efficiency of the algorithm in proposing and finding the samples with low misfit values. Table 2 lists the estimated parameters corresponding to the best model in different GP runs.

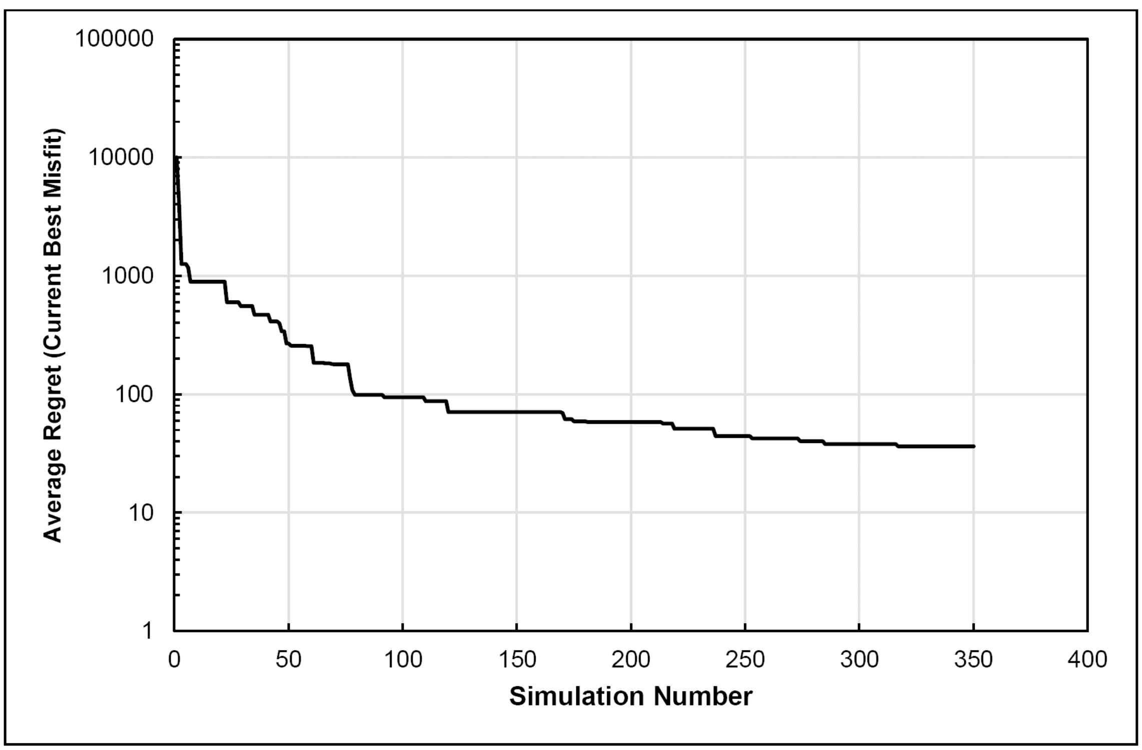


**Figure 3—Minimization of the regret function**

Table 2—Comparison of truth model values and the best four match results

| Parameter | Truth | Match 1 | Match 2 | Match 3 | Match 4 |
|---|---|---|---|---|---|
| kx Multiplier | 1.000 | 1.227 | 1.117 | 1.762 | 0.926 |
| ky Multiplier | 1.000 | 1.108 | 1.381 | 0.817 | 1.268 |
| kz Multiplier | 1.000 | 0.649 | 1.500 | 1.092 | 0.604 |
| F2 Multiplier | 0.500 | 0.454 | 0.021 | 0.000 | 0.137 |
| F3 Multiplier | 0.000 | 0.001 | 1.000 | 0.000 | 0.001 |
| F4 Multiplier | 1.000 | 0.511 | 1.000 | 0.864 | 0.330 |
| Aquifer Length, ft | 20000.000 | 27165.040 | 19549.130 | 27918.210 | 24320.680 |
| Aquifer Permeability, md | 500.000 | 812.587 | 1000.000 | 812.322 | 598.472 |
| $N_w$ Corey's exponent for $K_{rw}$ | 2.000 | 3.561 | 5.000 | 2.907 | 1.500 |
| $N_o$ Corey's exponent for $K_{ro}$ | 2.000 | 1.500 | 1.500 | 2.348 | 2.975 |
| $K_{ro}$ at $S_{wi}$ =0.25 | 0.900 | 0.885 | 0.890 | 0.950 | 0.950 |
| $K_{rw}$ at $S_{or}$ =0.15 | 0.500 | 0.490 | 0.600 | 0.520 | 0.530 |

The corresponding history matching results of the best model obtained from Match 1 (after 285 black oil simulation runs) are presented in Figure 4. Clearly, a reasonable match has been attained for all dynamic outputs of the subject wells.

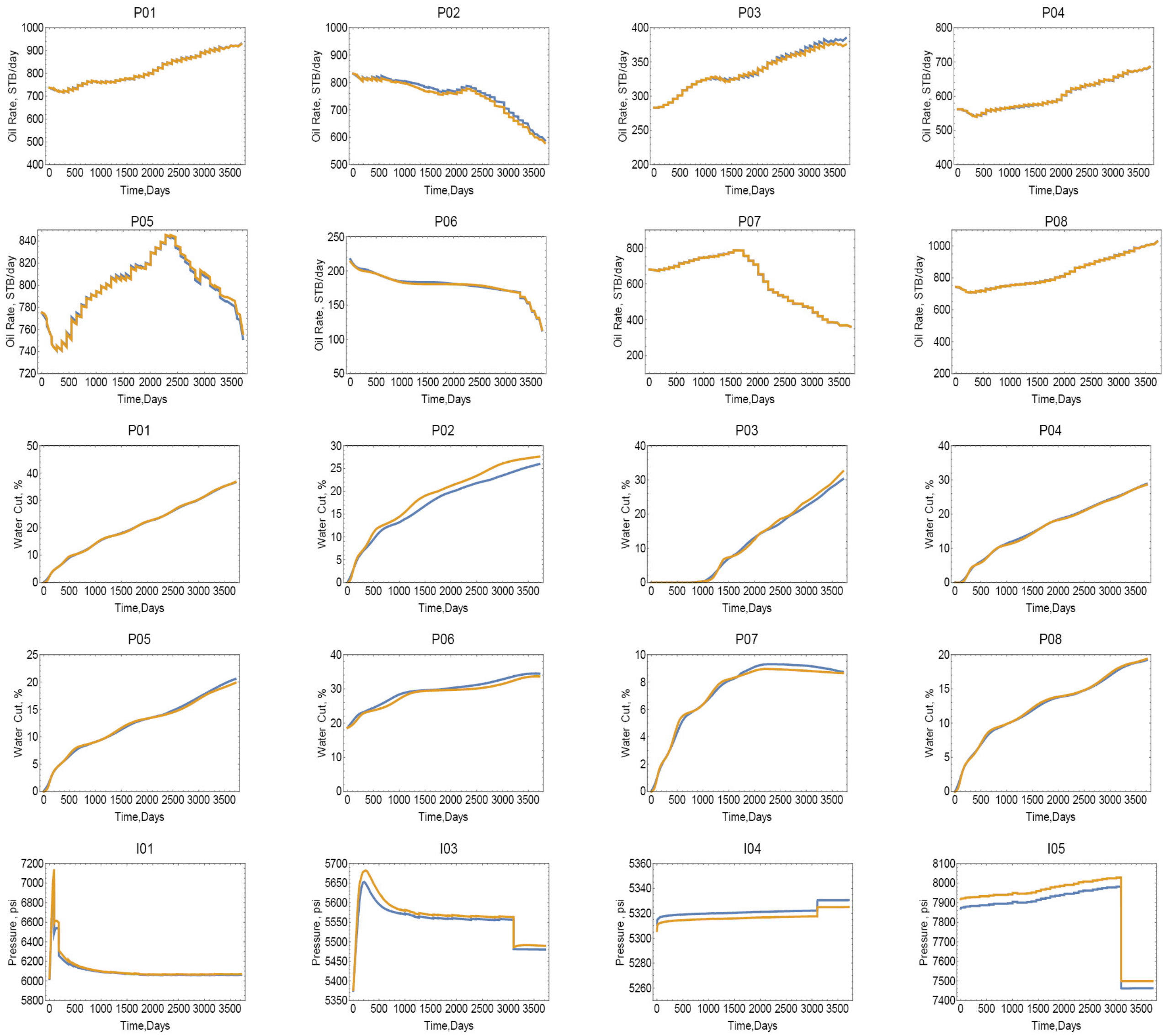


**Figure 4—Oil rate, water cut and pressure match results. The blue curves represent the truth model response and the orange curves are the simulated data**

The value of observed dynamic data in constraining the reservoir parameters is assessed by performing a Markov chain Monte Carlo (McMC) process (Shonkwiler and Mendivil, 2009; Kruschke 2010) in which the prior ranges and the likelihood of the sampled models is combined to estimate the posterior distributions. The posteriors are assessed using a Bayes rule as follows:

$$P(m|o) \propto P(o|m) \times P(m) \tag{3}$$

in which, P(m|o) is the probability of model parameters given the observed data (*a posterior*) that is proportional to P(o|m) is the likelihood and P(m) is the prior probability (*a prior*). The constant of proportionality is estimated using the Gibbs sampler. The numerical methods involved in the Gibbs sampling algorithm (Geman and Geman, 1984) requires frequent sampling and evaluation that is not generally feasible using the actual reservoir simulations due to computational cost. Therefore, similar to history matching algorithm, we use a validated GP proxy model to approximate the relationships between model inputs and output.

In this study, we combine the entire samples that we obtained in our four GP runs and create a pool of data. This relatively large pool of 1400 samples is used to construct a GP model by optimizing the proxy hyper-parameters (Tong, 2013). The obtained GP model is able to properly approximate the model behavior. The quality of such an approximation is judged by performing a statistical cross validation (CV) test. In our problem, a leave-out-one-cross-validation (LOOCV) is performed. Figure 5 shows the absolute prediction errors and the predictively of the GP proxy. In particular, most of the errors are close to zero (Fig. 5; left) and the data are approximately lied on a 45° diagonal line (Fig. 5; right) which indicates a relatively good prediction ability of the proxy.

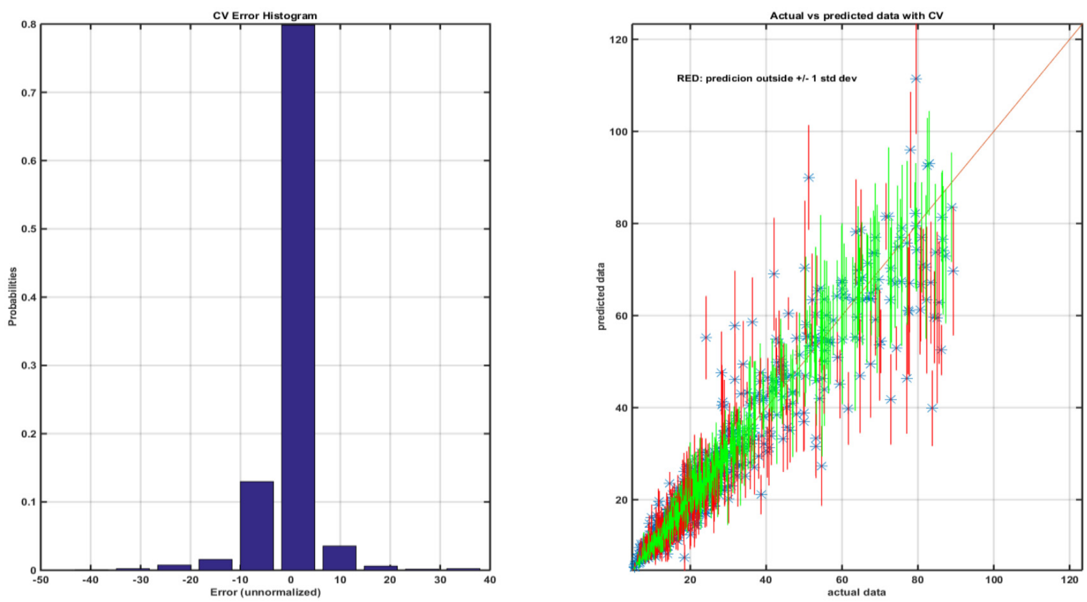


**Figure 5—Validation of the proxy model: The cross validation error (left) and the predictivity of the GP model (right). The green bars are the points that have the prediction and the actual values within a unit prediction error and the red bars are outside this range**

For this work, we use uniform priors defined over the possible ranges of variations as indicated in Table 1. The likelihood or P(o|m) is approximated by assuming a Gaussian likelihood. This is obtained from the misfit values using the following relationship (Erbaş and Christie, 2007):

$$P(o|m) \propto e^{-M^2} \tag{4}$$

The McMC is then performed on the constructed proxy and repeated dependent samples are drawn to directly sample from the posteriors. We performed a multi-chain McMC with 10 chains and 100,000 samples per chain (Tong, 2013). A similar number of burn-in samples are also evaluated and discarded to ensure the process is stable and is not stuck in sampling from low probability tails. Figure 6 shows a 1D projection of the posterior probabilities and indicates how the knowledge of posterior can help understand the value of information. The posterior probabilities are used for probabilistic prediction of the future reservoir performance.

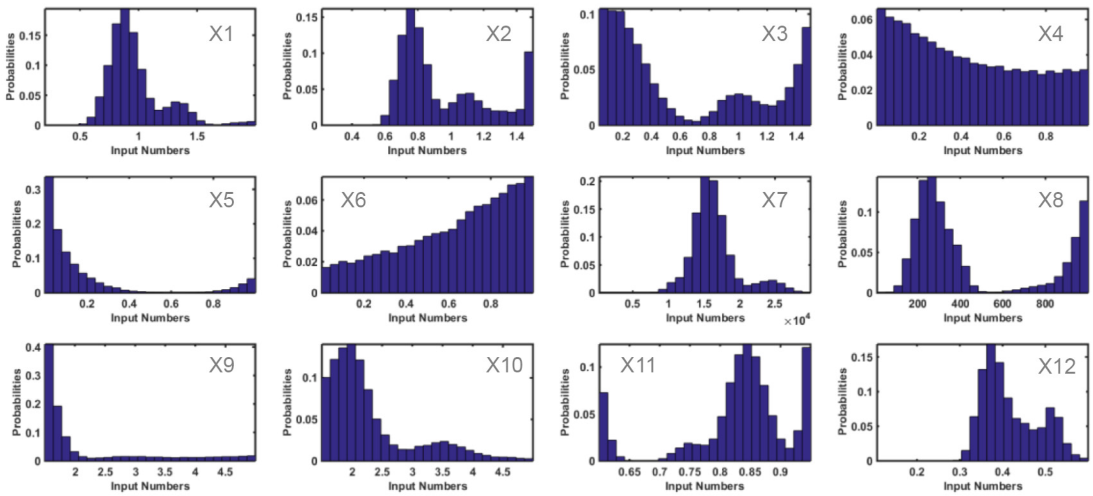


**Figure 6—Posterior distributions of the unknown parameters from Gibbs sampling**

## Conclusions

In this paper, we presented a new approach for history matching reservoir simulation models. A Gaussian Process surrogate model was used to efficiently approximate the relationships between the model parameters and the misfit values. A statistical criterion (i.e. Expected Improvement) was defined over the GP model to detect the areas with low misfit potentials. This approach was successfully tested for a realistic faulted fluvial reservoir model with 12 injectors and producers. The GP optimization was repeated a number of times and each time the results showed a consistent fast convergence (<350 simulations) for history matching of this 12-dimensional problem. The GP model fitted to all samples could provide a good predictivity with a low cross-validation error. This surrogate was implemented through a multi-chain Gibbs sampling technique and the posterior distributions were estimated. The GP modeling was found as a smart method that can provide promising results and an efficient and fast optimization approach for high dimensional problems.

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