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Rapid Reservoir Modeling: Prototyping Of Reservoir Models, Well Trajectories and Development Options using an Intuitive, Sketch-Based Interface

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Abstract
Constructing or refining complex reservoir models at the appraisal, development, or production stage is a challenging and time-consuming task that entails a high degree of uncertainty. The challenge is significantly increased by the lack of modeling, simulation and visualization tools that allow prototyping of reservoir models and development concepts, and which are simple and intuitive to use. Conventional modeling workflows, facilitated by commercially available software packages, have remained essentially unchanged for the past decade. However, these are slow, often requiring many months from initial model concepts to flow simulation or other outputs; moreover, many model concepts, such as large scale reservoir architecture, become fixed early in the process and are difficult to retrospectively change. Such workflows are poorly suited to rapid prototyping of a range of reservoir model concepts, well trajectories and development options, and testing of how these might impact on reservoir behavior.

We present a new reservoir modeling and simulation approach termed Rapid Reservoir Modeling (RRM) that allows such prototyping and complements existing workflows. In RRM, reservoir geometries that describe geologic heterogeneities (e.g. faults, stratigraphic, sedimentologic and/or diagenetic features) are modelled as discrete volumes bounded by surfaces, without reference to a predefined grid. These surfaces, and also well trajectories, are created and modified using intuitive, interactive techniques from computer visualization, such as Sketch Based Interfaces and Modeling (SBIM). Input data can be sourced from seismic, geocellular or flow simulation models, outcrop analogues, conceptual model libraries or blank screen. RRM outputs can be exported to conventional workflows at any stage. Gridding or meshing of the models within the RRM framework allows rapid calculation of key reservoir properties and dynamic behaviors linked with well trajectories and development plans. We demonstrate here a prototype of the RRM workflow using a number of examples.

The work is significant because it allows, for the first time, application of rapid prototyping methods in reservoir modeling and simulation. Such methods are widely used in other fields of engineering design and allow improved scoping of concepts and options prior, or in addition, to detailed modeling. Moreover, SBIM can be used on a range of hardware architectures, including table tops and surface PCs, fostering collaboration within integrated asset teams.

Introduction
Hydrocarbon reservoirs typically contain an array of complex geologic heterogeneities that are at, or below, the resolution of seismic data, so their geometry and spatial distribution is uncertain. These heterogeneities may be structural, stratigraphic, sedimentologic and/or diagenetic in origin, and often impact on flow behavior and hydrocarbon recovery; hence, they must be captured in reservoir models (e.g. Jones et al., 1994; 1995; Kjønsvik et al., 1994; White and Barton, 1999; Jackson et al., 2003; 2005; 2009; White et al., 2004; Matthäi et al., 2007a; Geiger et al., 2009; Sech et al., 2009; Choi et al., 2011; Deveugle et al., 2011). Reservoir modeling workflows have remained essentially unchanged for the past decade, facilitated by commercially available software packages such as Petrel and IRAP RMS (see Jackson et al., 2013 for an overview). These workflows begin with the construction of a geo-cellular reservoir model, in which a largely deterministic structural and stratigraphic framework is used to define the overall reservoir volume, and also compartments and zones within the reservoir. A grid is then constructed within each zone, typically using pillars that are continuous from the top to the base of the modelled volume, and

layers that may vary in thickness or be truncated by reservoir zone boundaries. Geostatistical methods are used to populate each grid cell with a geologic indicator (such as facies or rock type) and associated petrophysical properties. The resulting models typically contain several millions to tens of millions of cells, and may be upscaled onto a coarser grid prior to flow simulation. Despite its ubiquity, there are a number of shortcomings with this workflow, including (but not limited to):

1. Conventional modeling workflows are slow, often requiring many months from the development of initial model concepts to flow simulation or other outputs;
2. Conceptual geologic models become fixed early in the modeling process, with uncertainty explored using geostatistical methods within the framework of a single conceptual model, rather than across a range of possible geologic concepts;
3. It is difficult or impossible to rapidly explore a range of conceptual models, well trajectories and development options and test how these might impact on reservoir behavior;
4. The introduction of pillar grids early in the modeling workflow limits the spatial resolution of the model, the complexity of the geologic architectures that can be captured, and focusses modeling effort on population of grid blocks with rock properties using geostatistical modeling methods;
5. Geostatistical methods are often non-intuitive, and require inputs that are not closely linked to the underlying geologic concept, so it can be difficult for the geologist to create a digital version of the model concept;
6. Integration across different disciplines is made more difficult by the use of different software tools, and also by different model grid types and resolution (e.g. fine- versus coarse-grids for geologic and flow simulation models; unstructured meshes for geomechanical modeling).

The petroleum industry still lacks a rapid, intuitive approach to create conceptual reservoir models prior to embarking on a detailed study. The first stage of any model build is to draw it; ‘if you can’t draw it, you can’t model it’ (quote from Glyn Williams of BP, via Mike King of Texas A&M). Such drawings are currently created in two dimensions (2D) on paper, but would be better done digitally in three dimensions (3D) when they could be used as a direct model input and be stored electronically to document how the reservoir model concepts were evaluated. However, this requires a new software tool to allow 2D digital sketches in map or cross-section to be extrapolated rapidly and sensibly to 3D; it cannot be done using general drawing or computer-aided-design/computer-aided-manufacturing (CADCAM) packages which lack any geologic insight. The tool needs to ‘help’ the user to create geologically realistic 3D digital models. Moreover, the tool needs to allow rapid (close to real time) calculation of reservoir properties, to eliminate the poor connection between software to create models, and software to make calculations using models. Rapid calculation of reservoir properties such as fluid volumes in place, reservoir connectivity to and between wells, swept reservoir volumes, and simulation of pressure transient tests and seismic response, allow improved use of dynamic information to test and constrain geologic concepts, rapid testing of concepts against reservoir data, and real-time evaluation of reservoir behavior for different geologic concepts.

The aim of this work is to develop rapid reservoir modeling (RRM) software for prototyping of complex reservoir models, well trajectories and development options, by means of novel, sketch-based interaction and modeling coupled with exploratory visualization and close-to-real-time numerical analysis. The new approach does not replace existing workflows; rather, it supplements them by allowing rapid testing of geologic and development concepts and how these impact on reservoir behavior. The software is differentiated from existing workflows by the rapid and intuitive generation of reservoir and well geometries using sketch-based interfaces and modeling (SBIM). Very few previous studies have proposed a sketch-based approach to geologic model construction; those that have do not include the full suite of structural, stratigraphic, sedimentologic and diagenetic heterogeneity found in hydrocarbon reservoirs, or rapid meshing and computation of reservoir properties (e.g. Lemon and Jones, 2003; Lidal et al., 2012; Natali et al., 2014).

Prototyping versus detailed reservoir modeling

A significant difference between reservoir modeling, using the typical and ubiquitous workflow outlined above, and modeling in other engineering settings such as the automotive or aeronautical industries, is the comparative lack of a prototyping stage (Shah, 2001; Evans, 2003; Arisoy and Kara, 2014). When there is a large range of possible conceptual models, it is common in these industries to rapidly *storyboard* a number of model concepts and assess their performance using rapid calculations, perhaps with a simplified description of the relevant fluid dynamics or other key physics. This prototyping stage is undertaken prior to, and to assist in, selection of a smaller number of concepts for detailed modeling and analysis. Such an approach is rarely, if ever, adopted in reservoir modeling (e.g. Bentley and Smith, 2008), in part because of the lack of available software tools. Instead, a single conceptual model concept is selected (typically this fixes the depositional environment(s) and key structural and stratigraphic surfaces) which remains unchanged for the rest of the modeling process and in all model predictions.

Flexibility in modeling using conventional workflows is particularly restricted once a grid has been defined. All subsequent modeling steps involve populating the grid cells with parameter values using one or more of the numerous geostatistical methods developed for this purpose (e.g. Journel and Huijbregts 1978; Haldorsen and Damsleth 1990; MacDonald and Aasen 1994; Strebelle 2002). Changes to the conceptual model at this stage require the grid to be re-built, which is a challenging and time-consuming task. Consequently, geologic uncertainty is typically investigated by stochastically varying the parameter values assigned to grid blocks, rather than by changing the model concepts such as large scale structure and stratigraphic architecture; changing the latter requires a new grid to be built. Yet modern reservoir surveillance techniques such as 4D seismic often reveal new details of the reservoir architecture that may require updating of the model concepts rather than changes to the geostatistical properties (e.g. Huang et al., 2011; Huang and MacBeth, 2012). The variability in reservoir model behavior resulting from uncertainty in model concepts may be far larger than the variability arising from uncertainty in the geostatistical population of grid cell values. For example, Deveugle et al. (2014) showed that the geostatistical method used to populate grid cells with facies and petrophysical properties in a fluvial-deltaic reservoir analogue had a much smaller impact on predicted reservoir behavior than the initial choice of conceptual model and correlation scheme. Moreover, the model inputs required for the geostatistical methods were largely non-intuitive and difficult to extract from the data available to the geologist.

Rapid prototyping requires a software tool that allows reservoir conceptual models, and also well trajectories, to be rapidly created using a simple and intuitive method, with some key reservoir properties calculated rapidly. Such a tool is proposed here, to allow development concepts to be tested against a range of reservoir concepts in a prototyping stage prior to detailed modeling using conventional workflows. The rapid modeling approach is based on sketching of geologic surfaces and well trajectories, using tablet machines with touch-sensitive screens or equipped with a stylus.

Surface-based reservoir modeling

A key aspect of the RRM workflow is that all geologic heterogeneity of interest, regardless of whether it is structural, stratigraphic, sedimentologic or diagenetic in origin, is represented using surfaces *prior* to generating a grid. Each surface denotes either:

1. A fault surface;
2. A stratigraphic surface;
3. A boundary between facies associations, and/or facies types within facies associations, and/or rock types or lithologies within facies types;
4. A boundary between different regions of diagenetic modification of rock properties;
5. A fracture surface.

The surfaces are defined and ranked in a hierarchy that uses relationships to specify which surfaces truncate, are truncated by, or conform to, other surfaces (e.g. Fig. 1). This hierarchy depends upon the geology being modeled (see, for example, Kjønsvik *et al*., 1994; Jones *et al*., 1995; Sech *et al*., 2009; Choi *et al*., 2011). The software ‘helps’ the modeller create geologically realistic surfaces by having these and other rules embedded to control surface interactions (e.g. Table 1; Fig. 2).

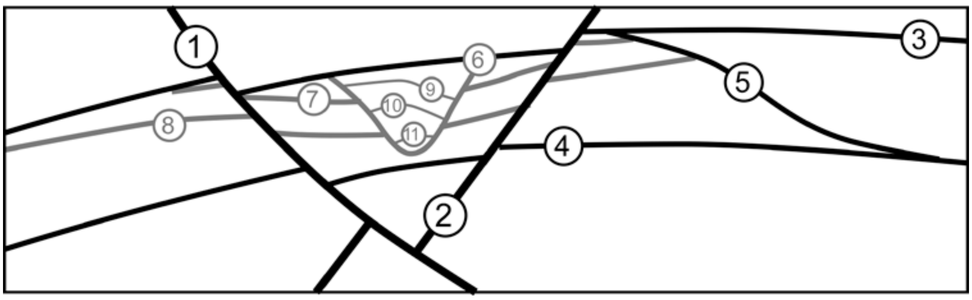

Figure 1: Example surface hierarchy. Fault surfaces (1) and (2) are at the top of the hierarchy; surface (1) truncates and offsets all other surfaces; surface (2) truncates and offsets all surfaces except surface (1). Stratigraphic surfaces (3), (4) and (5) are at the next level of the hierarchy; surface (3) truncates surface (5) and surface (5) conforms with surface (4); all surfaces lower in the hierarchy are truncated. Facies association boundaries (6), (7) and (8) are at the next level of the hierarchy; surface (6) truncates surfaces (7) and (8). Facies boundaries (9), (10) and (11) are at the lowest level of the hierarchy. The hierarchy is a generic example and is deliberately shown with no scale. Modified from Jackson et al., 2013.

Using surfaces to model geologic heterogeneity is, in principle, simple: *any* geologic heterogeneity, which impacts on the spatial distribution of petrophysical properties, is modeled as one or more discrete volumes bounded by surfaces. Within these discrete volumes (termed ‘geologic domains’), the petrophysical properties are constant. This is equivalent to a grid-based approach to reservoir modeling in the sense that petrophysical properties within grid cells are constant. However, in our approach, petrophysical properties are constant within geologically meaningful domains, rather than cells of arbitrary size and shape. The approach offers distinct advantages in allowing reservoir modeling using SBIM. It is natural to draw conceptual models using surfaces: when asked to create a representation of a reservoir, geologists do not produce a sheet of graph paper and start colouring in the squares to mimic geostatistical modeling on a grid. Rather, they produce 2D plan- and cross-section-view sketches in which key features are represented by surfaces. The use of SBIM allows this approach to be extended to create digital models in 3D. The models thus created are ready for meshing and rapid calculation, or export to conventional workflows. The models can also be stored to illustrate the decision-making that has led to the creation of a specific reservoir (simulation) model.

Petrophysical properties within the geologic domains are required for calculation of reservoir properties and these may be determined from core- or log-derived data, or upscaled from smaller-scale models that capture the impact on flow of heterogeneities below the resolution of the modeled surfaces. Similar approaches are used to assign petrophysical properties in conventional, grid-based models (e.g. Pickup *et al*., 1994; Kjønsvik *et al*., 1994; Jones *et al*., 1995). Ideally, properties that represent the smaller-scale heterogeneities are derived at the Representative Elementary Volume (REV) for a given small-scale heterogeneity, in which case they represent effective properties that can be assigned to rock volumes in the model regardless of their size (e.g. Renard and de Marsily, 1997; Jackson *et al*., 2003; Nordahl and Ringrose, 2008). However, it may not always be possible to identify the REV. In this case, upscaled properties represent equivalent properties that are sample volume dependent (Renard and de Marsily, 1997). Nevertheless, using these equivalent properties in larger-scale models is better than omitting the smaller-scale heterogeneity entirely. These issues are identical to those faced when building conventional, grid-based models, where grid-cell volumes rarely correspond to the REV for the smaller-scale heterogeneities they contain.

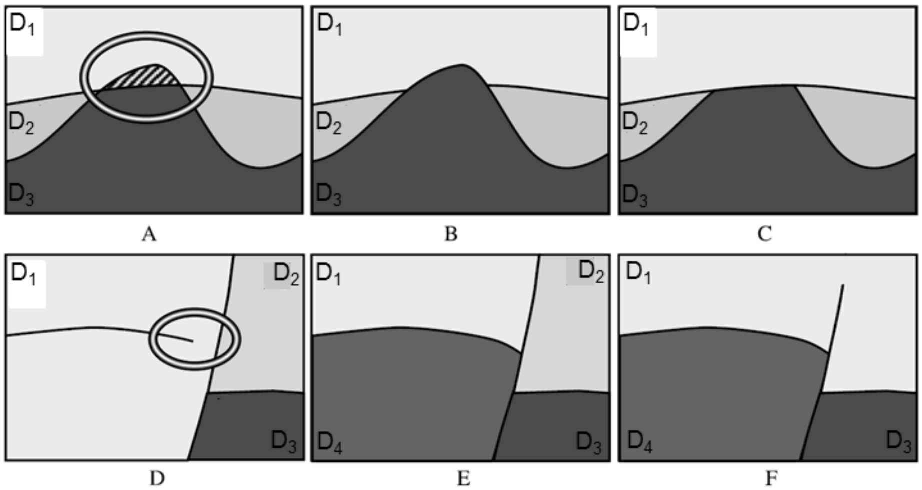

Figure 2: Typical rules embedded in the RRM software. To model these geologic domains (D_1, D_2 and D_3) only scenarios B, C, E, and F are allowed. In A, the two geologic domains D_3 and D_1 overlap which is forbidden; in B, D_2 and D_3 onlap onto (or are intruded by) D_3, while in C, D_2 onlaps onto (or is intruded by) D_3, and both are erosionally truncated by D_1. Both of these latter scenarios are allowed. In D, domain D_1 extends around a discontinuous stratigraphic or sedimentologic boundary which is not allowed; in E, the boundary is properly truncated by a fault to create a new domain D_4, while in F, the fault tips out in domain D_1. Both of these latter scenarios are allowed. Modified from Caumon et al. (2009).

Table 1: Example rules for the interaction of fault surfaces and stratigraphic surfaces. This list of rules is not exhaustive.

All surfaces/domains:
1. Cannot self-intersect
2. Cannot be adjacent to themselves
3. Can only exist once in the stratigraphy
4. Domain *i* cannot occupy the same volume as domain *j* ($i \neq j$) (e.g. Fig. 2A)
Fault surfaces:
1. Truncate and offset older surfaces (e.g. Fig. 1, Fig. 2D-F)
2. Normally monotonic
Stratigraphic surfaces:
1. Erode or conform to older stratigraphic surfaces (e.g. Fig. 1, Fig. 2B,C)
2. Normally monotonic
3. Must always define closed domains (e.g. Fig. 2D)

Sketch-based interfaces and modeling (SBIM)

Creating 3D objects in a 3D environment may require hours of work and specialized training. To alleviate this problem, sketch-based interfaces and modeling (SBIM) attempts to mimic the natural way people communicate ideas through 2D drawings and translate them into 3D models. One of the main challenges in SBIM is to reduce the ambiguity in 3D modeling from 2D sketches. This is achieved in RRM using 2D sketches in both map and cross-sectional views and extrapolation of surfaces constrained by geologic rules. Existing 3D geologic modeling approaches do not take advantage of interface approaches that users can intuit naturally, allowing geologists to rapidly create conceptual models; instead they rely on the 'windows-icons-menus-pointer' (WIMP) paradigm. Such WIMP-based tools are typically non-intuituve and often require specialized training, placing a barrier between the geologist developing the concept, and creation of a digital model of the concept. RRM is an integrated system that mimics the pen and paper methods used by geologists. It combines map and cross-section sketches created using SBIM to produce 3D geologic models. The system interprets sketches and, under geologic constraints, generates complex multilayer 3D models from the 2D sketches. The system automatically identifies the age of the rock layers using the principle of superposition, and the contact relationships and interaction rules defined for the surfaces; this reduces the tedious task of manually defining the stratigraphic layering, supporting the idea of rapid prototyping and avoiding easy-to-make mistakes. Moreover, geologic rules are imposed directly on the sketch, enabling the creation of valid geologic models.

Outside of the geologic modeling literature, numerous studies have explored SBIM to a provide more intuitive method to create 3D models (e.g. Igarashi et al. 1999; Cherlin et al. 2005; Nealen et al. 2007; Orbay and Kara, 2012). For example, Olsen et al. (2011) combined image-assisted SBIM of 3D objects with sketch-annotations. Such annotations are iconic symbols to indicate modeling operations on the object being sketched. However, in RRM we mimic how geologists sketch using geologic rules to constrain surface geometries and interactions, rather than creating our own sketching language. The RRM software builds on earlier studies in which SBIM was applied to create geologic models using specific data and/or operations (Amorim et al., 2012; 2014). Amorim et al. (2012) presented a SBIM system for modeling geologic horizons, which provides a set of sketch-based operators to help geologists to edit, model and augment horizons. However, their system focused primarily on the augmentation of previously extracted (using a semi-automatic method) horizons. To create new horizons from 'blank screen' the approach was limited to the creation of an initial Coons surface (Salomon, 2005) from four lines, one sketched on each face of a seismic cube. This initial surface usually needs a careful and time-consuming process of augmentation to obtain the final desired shape. Amorim et al. (2014) presented a SBIM system for creating 3D geologic models using sketch-annotations applied in 2D map view; information to constrain the model in cross-section is applied through the use of annotations on the map. The SBIM operators developed here are more robust and flexible than these earlier approaches, allowing rapid creation of surfaces constrained by sketches in both map and cross-section views.

The Rapid Reservoir Modeling Workflow

The RRM workflow comprises 8 key steps, summarized in Figure 3. ***Step 1*** comprises data: in this initial version of RRM, reservoir geometry can be created / edited with SBIM methods from the following input data:

1. Surfaces imported from commercial geo-modeling tools (e.g. seismic horizons or entire geomodels).
2. Reservoir geometry created from a blank screen; this functionality allows conceptual models to be rapidly created and their reservoir behaviour investigated.
3. Reservoir geometry created using template surfaces representing common geologic architectures that can be used to rapidly create geologic features with similar generic shapes (e.g. clinoforms). The surfaces can then be manipulated in step 2 to vary the initial template geometry. Users can also create their own template surfaces.

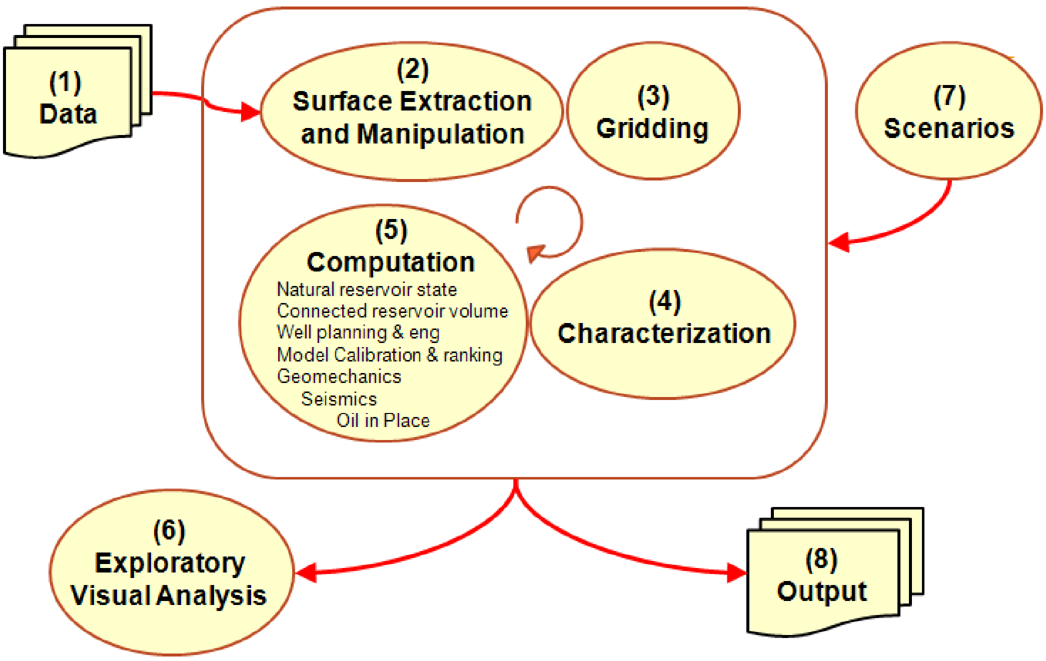

Figure 3: Rapid Reservoir Modeling (RRM) framework with its 8 main stages and components

Step 2 comprises a set of SBIM operators that allow imported or created surfaces to be modified and combined, resulting in new reservoir geometry. Users can also sketch and model simple wells, represented as 3D curves. ***Step 3*** is discussed further below. ***Step 4*** comprises characterization, in which a user-expandable library of default petrophysical property values (permeability, porosity, Young's modulus, density etc.) for given rock types is available and geologic domains are populated using SBIM to select regions combined with 'drag and drop'. This is not a geostatistical approach; instead, rock types and corresponding, single-value petrophysical properties are directly assigned to geologic domains defined by the surfaces created in step (2). Geostatistical methods to populate petrophysical properties within the RRM framework may be added in a future phase of the project. However, an interface between RRM and commercial geo-modeling tools (see step 8) allows surface-based models constructed using all or part of the RRM workflow to be exported and the geostatistical methods available in these commercial packages to be applied if this is required.

Steps 3 and 5 comprise gridding and computation; these steps are discussed in greater detail in the next section. ***Step 6*** comprises exploratory visual analysis in which reservoir geometry, property distribution, and simulation results can be visualized on desktop computers and interactive display surfaces. This step allows integration across disciplines with a shared interface allowing interrogation of key geologic and engineering parameters. ***Step 7*** comprises scenarios: a global database and Wiki where geologic scenarios can be collected, stored, and accessed for analysis and modification. Geologic scenarios are often lost in conventional workflows, existing only in the head or on paper; the RRM workflow allows 3D conceptual models to be rapidly created and then stored for future use. ***Step 8*** comprises output: the new or modified reservoir geometry can be exported to commercial geo-modeling tools for future use in conventional reservoir modeling and simulation workflows. Output includes (a) the surfaces defined at the end of step (3), converted to the required format, and (b) corner-point grids and associated rock properties defined at the end of steps (3) and (4).

Rapid gridding and calculation of static and dynamic reservoir behavior

An important aspect of RRM is to understand how different geologic model concepts impact static and dynamic reservoir properties. This understanding helps to rank and cluster different model concepts and capture model uncertainty more accurately. While it is possible, in principle, to export the models developed in RRM to standard geomodeling tools and perform the required calculations, such a workflow would be comparatively slow and at odds with the fundamental idea of RRM, which is to *rapidly* prototype different geologic model concepts. Hence the essential static and dynamic calculations must be possible within the RRM framework. Static reservoir properties include oil and gas in place, Lorenz coefficients, pressure distribution, stress state and, in the case of high-pressure-high-temperature (HPHT) reservoirs, temperature distributions. Full-physics simulations based on Black-Oil or compositional models are too time-consuming. Proxy calculations are therefore needed to obtain rapid feedback on how the dynamic reservoir behavior could change across a suite of different geologic model concepts. Proxies for dynamic reservoir behavior include dynamic Lorenz coefficients and flow capacity curves (Shook and Mitchel, 2009), streamline simulations (Thiele, 2005; Datta-Gupta and King, 2007), geologic well-testing (Corbett et al., 2012; Chandra et al., 2013; Agada et al., 2014), or flow diagnostics that provide swept reservoir volumes, flow partitioning, and well-allocations based on static tracer distributions and time-of-flight calculations (Moyner et al., 2014).

All these computations allow us to approximate static and dynamic reservoir behaviors but require, with the exception of fluids in place and Lorenz coefficients, the solution of a steady-state Laplace (i.e. elliptic) equation. Regardless of the method that is employed to discretize an elliptic equation, the resulting linear system of equations can be inverted efficiently using algebraic multi-grid methods (AMG). Since the CPU time of AMG scales as $N\log(N)$ where N is the number of unknowns, even large systems of equations can be solved efficiently (Stüben, 2007), which enables almost real-time feedback on changes in the static and dynamic reservoir behavior when different geologic model concepts are explored with RRM. However, one fundamental challenge remains in this approach and needs to be explored: All calculations require the generation of a grid on which the petrophysical properties can be distributed and the flow equations can be discretized. Modeling the petrophysical properties in RRM has been described above; petrophysical properties are assumed to be constant within a given reservoir volume, unless a pre-existing reservoir (simulation) model with a given (geostatistical) property distribution is modified.

There are several approaches to create the grid for discretizing the flow equations. As a first step, basic pillar-gridding technologies can be used to automatically translate the surfaces into a corner-point grid and use standard finite difference methods with the two-point flux approximation (TPFA) for flow computations. This also enables straightforward export to commercial simulation and modeling software. In addition, cells in a corner-point grid are akin to hexahedral finite elements so standard Galerkin finite element methods with quadratic basis functions can be used to facilitate mechanical simulations. This approach is appropriate for relatively simple reservoir geometries. However, pillar-gridding, corner-point grids and the TPFA are unsuitable for structurally complex reservoirs, because numerical errors can be significant in these cases (e.g. Wu and Parashkevov, 2009; Lie et al., 2012, Jackson et al., in press) unless multi-point flux approximations are used (e.g. Aavatsmark, 2002). To limit the amount of computationally costly multi-point flux approximations, immersed boundary methods could be used such that the grid remains structured (and the TPFA is valid) in most of the reservoir except in the vicinity of the surfaces (e.g. Ahmadi et al., 2013).

A more natural approach to discretize complex reservoir geometries are tetrahedral finite element grids (e.g. Matthäi et al., 2007b; Milliotte and Matthäi, 2014, Jackson et al., in press; see Figure 4). A number of excellent libraries to generate tetrahedral finite element grids for complex reservoir geometries already exist, such as the Open Source CGAL library (http://www.cgal.org). While finite element techniques are standard in computational fluid techniques (e.g. Chung, 2010), their use in reservoir simulation is normally restricted to research-grade simulators (e.g. Matthäi et al., 2007a,b; Kolditz et al., 2012; Jackson et al., in press). The main argument is that finite element methods are not competitive from a computational point of view. While this may be true for a full-physics reservoir simulation, the argument does not apply to RRM, because only a very small number of finite element calculations to solve elliptic or quasi-elliptic problems are required. As noted above, the system of linear equation that results from the finite element discretization of an elliptic equation can be solved efficiently with AMG methods (Stüben, 2007).

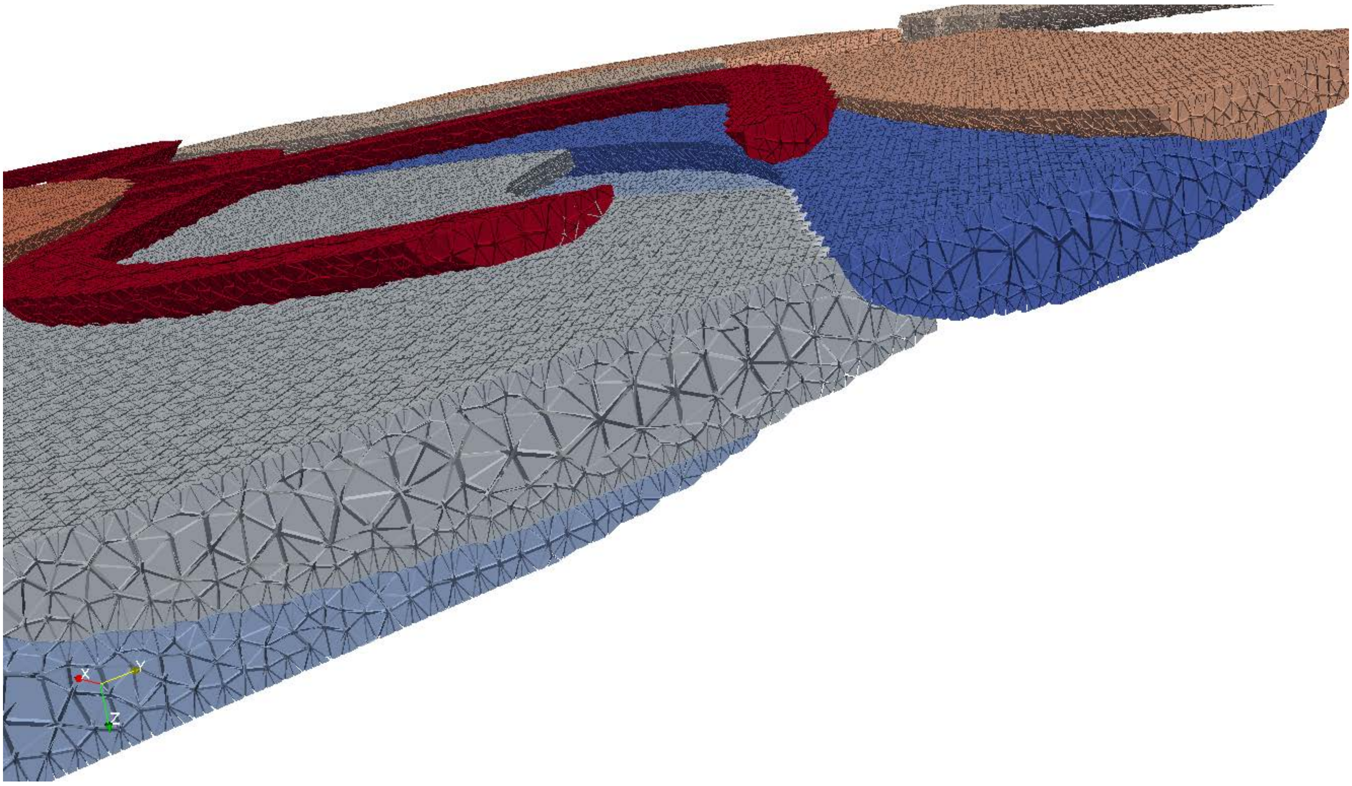

Figure 4. Tetrahedral finite element grid representing fluvial channels that have been modelled using a surface-based approach. Modified from Jackson et al. (2013).

One difficulty that remains when using finite element discretization techniques for rapid flow diagnostics is that mapping streamlines for time-of-flight calculations on unstructured finite element grids is difficult and requires special numerical treatment (Matringe et al., 2008). A remedy to this problem has been suggested by Shahavali et al. (2012) and Moyner et al. (2014), who use an operator identity to cast the time-of-flight definition along streamlines to a time-of-flight definition in Eulerian coordinates; that is, the time-of-flight can be computed directly on any unstructured grid by solving a steady-state advection equation. Once the time-of-flight has been obtained on the grid, dynamic Lorenz coefficients, flow capacity curves and travel times between injector and producer wells can be readily obtained. If the time-of-flight calculation is augmented by a steady-state tracer calculation, flow partitioning in the reservoir, swept reservoir volumes, and well-allocation factors can also be obtained at negligible extra computational cost (Moyner et al., 2014), which provides a full suite of fast flow diagnostic tools to explore, rank, and cluster different reservoir model concepts in terms of their approximate dynamic behavior in RRM.

Prototype implementation of SBIM for geologic sketches

In this paper we report a prototype implementation of SBIM for geologic sketches incorporating a graph-based representation of geologic surfaces. The graph is used to identify erosional stratigraphic surfaces, the stratigraphic series a rock layer belongs to, rock-layer adjacencies, and ensure compliance with geologic rules (Table 1). An alternative to this graph-based approach would be a rasterization of the sketches (Olsen et al. 2011); however, rasterization poses a problem when sketching contacts with different levels of detail, since very high resolution is required for a single fine detail. When a surface is sketched in 2D it is first super-sampled by equidistant points based on a zoom factor, to take into account the level of detail. Then a reverse Chaikin filter (Samavati and Bartels, 2004) is iteratively applied to remove noise from the input. If the new curve is valid after filtering, it is used to update the surface graph. A curve is invalid if it is outside the map, has self-intersections, or has only one intersection with the graph. Every new valid curve generates a closed region that represents a visible part of a rock layer in map view, and the curve belongs to the adjacent rock layer. There are two possible ways to update the graph depending on the number of intersections with the curve. If there is no intersection, the curve is converted into a new connected component of the surface graph; however, if there are 2 or more intersections, the segment between the two first intersections is added to the connected component of the graph intersected. Each vertex of the graph representation is described by its 2D position on the map, a list of neighboring rock layers, and an id that references the rock layer that the vertex belongs to. Using the vertices' ids and list of neighbors, the geologic rule that forbids a rock layer to be adjacent to itself is checked. To decide if a closed region corresponds to an erosional surface, we search for T-junction vertices and check their adjacent vertex ids. Based on these contacts we build a stratigraphic series composed of surfaces and rock layers. This series represents the stratigraphic order of surfaces and rock layers, and allows checking of the geologic rule that a rock layer can only exist once in the stratigraphy. Figure 5 illustrates an example of a contacts graph and its corresponding stratigraphy.

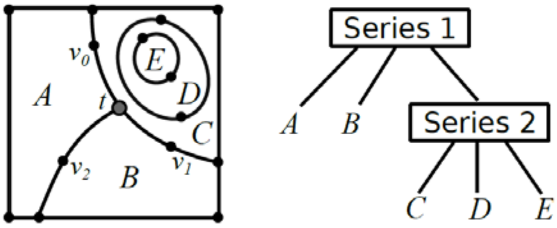

Figure 5: Illustration of contacts graph and corresponding stratigraphy. Vertex *t* has id = B and neighbors A and C. Vertex v_0 has id = C and neighbor A; v_1 has id = C and neighbor B; v_2 has id = B and neighbor A. Checking the T-junction *t* adjacency, the surface containing v_0 and v_1 is defined as erosional since their ids are equal. Modified from Amorim et al., 2014.

The final 3D geologic model comprises a combination of implicit geologic surfaces representing the interface between two geologic domains defining solid objects. The final solid objects are constructed using Constructive Solid Geometry (CSG) based on the age of the domains. Each surface is represented by an iso-value of a function F that interpolates geologic contacts and dip direction. A stratigraphic series is represented by a single function F and, within this series, geologic surfaces are represented by different iso-values, chosen based on domain age, ranging from 0 (surface on top of the youngest domain) to n (surface on top of the n^{th} domain). To automatically identify the stratigraphic order of domains, we extract their adjacency from the contacts graph. However, from this adjacency alone it is not possible to decide whether the sequence is in the correct or inverse age order. To resolve this ambiguity, we determine whether a point p inside a region within the series is on a syncline or an anticline. We check this by interpolating an implicit surface using the surface information defining this region. Since we do not yet have the rock layer order information, we arbitrarily interpolate the contacts at this time using zero as the iso-value; then, if $F(p) < 0$, the feature is an anticline, otherwise it is a syncline. In this preliminary work we chose the Hermite-Birkhoff Radial Basis Function (HBRBF) interpolation (Pereira et al. 2011) as the function F. HBRBFs create implicit surfaces that can interpolate values, gradients, or values and gradients at a point in space. Faults are defined by lines that represent the location of the truncation of other surfaces. Each fault is represented by its own HBRBF, which interpolates the fault line and the dip angle by including samples with value and gradient. We can define whether a point is on the left or right of the fault by evaluating this HBRBF function. Based on the type of fault, we create a displacement field for points on the left and another for points on the right. To create the 3D solid model of each layer, we use the automatically computed hierarchy of rock layers and surfaces and the HBRBF defining each surface. Each rock layer defines a 3D volume that will be combined with the others by CSG Boolean operations. To create a rock layer i we subtract the implicit surface with iso-value i from the one with iso-value i-1. Erosional surfaces are created using the series tree, where we subtract each entire series from its parent. Finally, we use the marching tetrahedra method (Treece et al. 1999) to extract the iso-surface of each rock layer.

Results from the RRM prototype

The RRM prototype described above was implemented on the Microsoft Surface Pro pen enabled tablet (Intel® Core™i5-3317U CPU with 1.7GHz, 4GB RAM memory, and Intel® HD Graphics 4000 graphics card). For all models shown here, the prototype enabled creation and visual inspection of 3D geologic models in real time using SBIM. To create the 3D models, using a marching tetrahedral sampling of $50 \times 50 \times 18$, the system took from 0.4s to 2.8s depending on the complexity of the model. However, to generate the figures in this section in high quality, we used a sampling of $200 \times 200 \times 72$, which took from 24s to 158s. To facilitate inspection of the interior of the 3D model, we implemented an exploratory visualization SBIM tool to cut the model, by sketching a line in map view. Our first results demonstrate that the RRM prototype is able to reproduce some basic geologic structures. Figure 6 demonstrates three types of fold, while Figure 7 shows normal and reverse faults. Figure 8 shows the sketching of a geologic map containing an erosional contact, and the automatically computed stratigraphy. In the first step, the sketched map contains two different rock layers within the same stratigraphic series. The age of the rock layers is ambiguous so we cannot generate a 3D geologic model. In the second step, a new surface is added that intersects with an existing one. The intersected surface then becomes an erosional contact and a new, younger geologic series is created. The respective geologic series and their rock layers are displayed hierarchically from older (bottom) to younger (top). In the final step, additional rock layers are inserted and dip information is provided. Based on the sketched geologic map the final 3D geologic model is created composed of two geologic series. As expected, the younger stratigraphic series erodes the older one.

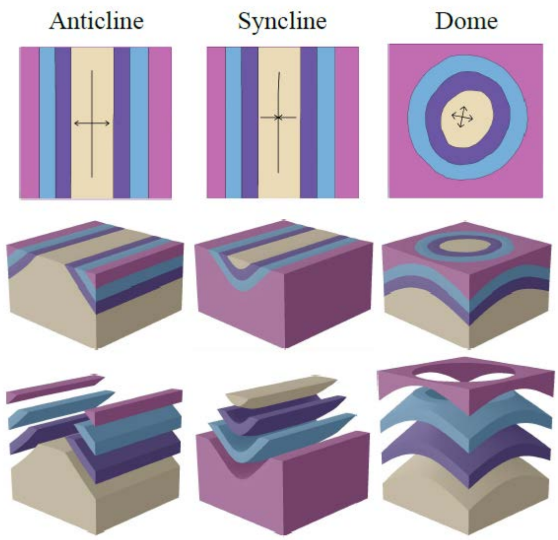

Figure 6. Sketch-based creation of basic fold structures in 3D. The upper row denotes a sketched geologic map in planview; the middle row shows the resulting 3D geologic model, and the lower row shows an exploded view of the same model. Modified from Amorim et al., 2014.

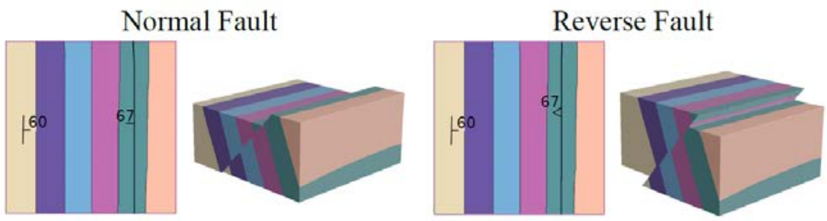

Figure 7. Sketch-based creation of basic normal and reverse fault structures in 3D. Each fault model is shown with a planview geologic map sketch and resulting 3D model; strike and dip information for the strata and faults are shown in planview. Modified from Amorim et al., 2014.

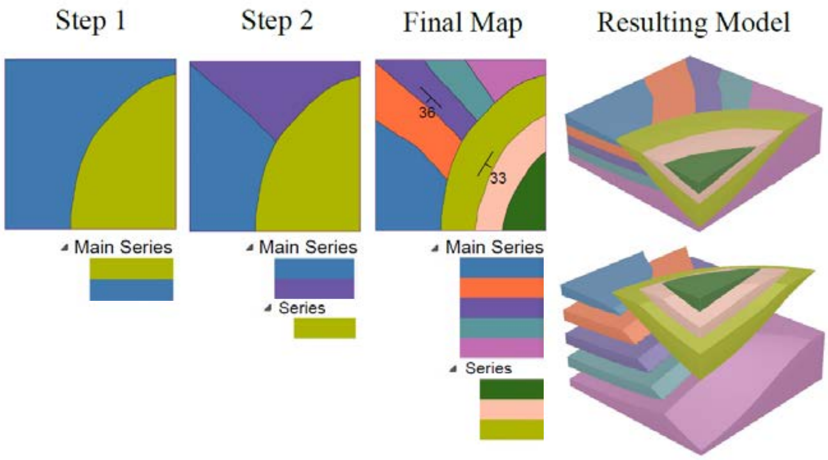

Figure 8. Erosional surfaces and stratigraphic series. In the first step the sketched map contains a single surface that separates two rock layers. In the second step a new surface is created that intersects the existing one, defining an erosional contact and creating a new stratigraphic series. The final map contains the two stratigraphic series with different rock layers. Modified from Amorim et al., 2014.

Finally, in Figure 9 we create a complex model combining multiple rock layers. In Figure 9(a) the sketched map contains two stratigraphic series separated by an erosional contact, and six dip-and-strike symbols. The 3D geologic model generated is displayed as a block model and as an exploded view with vertical exaggeration. In Figure 9(b) a reverse fault is sketched and in (c) we sketch a cutting line and the interior of the model is inspected. Finally, in Figure 9(d) another fault is sketched and the final 3D geologic model is presented using the same cut as before. As demonstrated in this section, the prototype RRM software is able to reproduce basic 3D geologic structures and more complex 3D geologic models based on rapidly created 2D sketches. The system enforces compliance with geologic rules, creating a valid geologic model.

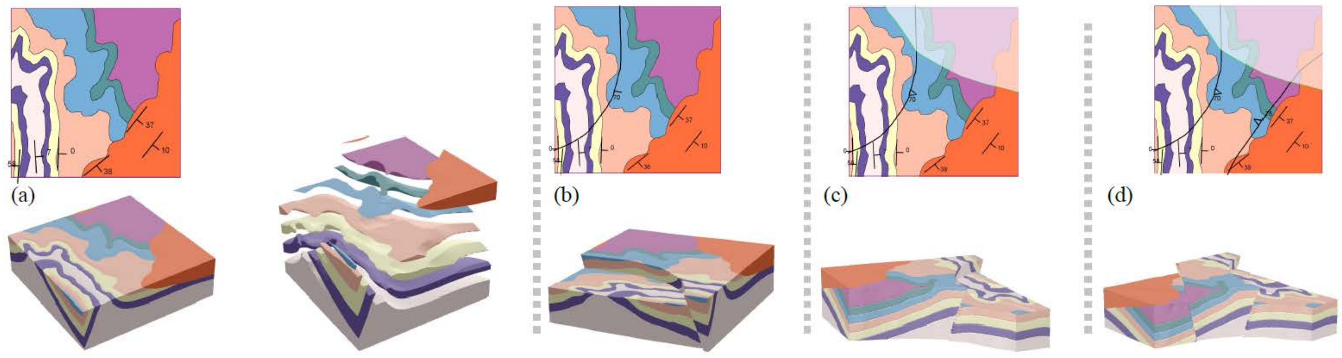

Figure 9. Combining more complex geologic contacts. In (a) the geologic map contains two different stratigraphic series. A fault is sketched and presented in (b). In (c) the interior of the same model is inspected by sketching a cutting curve. A new fault is sketched in (d) and a final 3D geologic model is presented using the previous cut. Modified from Amorim et al., 2014.

Discussion

RRM is not a finished product; work is ongoing to implement the full functionality described here. However, the approach already allows rapid prototyping and multi-purpose testing of 3D geologic scenarios (from outcrop to field and basin scale) constructed from sparse data using SBIM, employing an intuitive user interface that encourages natural collaboration of interdisciplinary teams of geologists, engineers, and geophysicists. Functionality currently under development includes full interfacing and leveraging with existing workflows and industry-standard commercial modeling packages, and real-time integration of reservoir modeling and characterization with automated analysis of essential dynamic patterns (flow, mechanics, seismics). The advantages of a tool such as RRM are manifold: it offers a (much) faster turnaround time compared to existing workflows, allows uncertainty in reservoir modeling to be better characterized at the appraisal and early development stage through rapid modification of reservoir geometry and characterization, provides a simple and intuitive approach to reservoir model construction and scenario testing, can capture complex geologic architectures using a surface-based approach and unstructured meshing for calculation of key reservoir properties and behaviors, and can act as a geometry hub feeding seismic, geomodel, geomechanical and flow models with common geometries and mesh. Moreover, it allows conceptual models and geometries to be created and retained in digital format, including a global database and Wiki of geologic scenarios and petrophysical properties. The approach addresses many of the shortcomings in conventional workflows, which are increasingly being identified as a barrier to progress (e.g. Agar et al.; 2013; Agar and Geiger, 2014).

Conclusions

We present a new reservoir modeling and simulation approach termed Rapid Reservoir Modeling (RRM) that allows, for the first time, application of rapid prototyping methods in reservoir modeling and simulation. Such methods are widely used in other fields of engineering design and allow improved scoping of concepts and options prior, or in addition, to detailed modeling. RRM is intended to complement, rather than replace, existing workflows. In RRM, reservoir geometries that describe geologic heterogeneities (e.g. faults, stratigraphic, sedimentologic and/or diagenetic features) are modelled as discrete volumes bounded by surfaces, without reference to a predefined grid. These surfaces, and also well trajectories, are created and modified using intuitive, interactive techniques from computer visualization. Input data can be sourced from seismic, geocellular or flow simulation models, outcrop analogues, conceptual model libraries or blank screen. RRM outputs can be exported to conventional workflows at any stage. Gridding or meshing of the models within the RRM framework allows rapid calculation of key reservoir properties and dynamic behaviors linked with well trajectories and development plans. We demonstrate here a prototype of the RRM workflow using a number of examples and confirm that complex 3D models of common geologic features can be created rapidly (within a few minutes) and intuitively using a sketch-based interface and commonly available tablet hardware.

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