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Calibrating Multi-Point Geostatistical Models Using Pressure Transient Data

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Abstract

Transient well test data conveys significant information about the subsurface heterogeneities in terms of some variations in the well test pressure response curves. It is therefore important to enhance the use of the well test data for building a validated geological model to include the effective reservoir heterogeneities that are reflected on the well test plots. In this work, we present a novel geoengineering workflow for geologically consistent updating of the geostatistical facies models using pressure transient data.

We use Multi-Point Statistical (or Geostatistical) simulations (MPS) with conditioning hard and soft data to generate the geostatistical realizations that can preserve the spatial connectivity of the facies. Static model transient tests are then generated using high resolution numerical simulations. The results are compared with the measured well test data for an inversion. The inversion step involves a geologically consistent Probability Perturbation Method (PPM) for perturbing the geostatistical models which are combined with a Gaussian Process (GP) modeling approach for finding the optimum spatial distribution of the facies and the other unknown model parameters. Conditional two-dimensional models of a low-energy anastomosing channelized model are considered in this study. The results show that using such an approach the spatial variation of the facies is maintained and the transitions across the facies boundaries are consistently preserved. In this paper, the geostatistical models are updated simultaneously with other unknown model parameters, including the PPM's parameter (r), facies permeabilities and the non-Darcy D-factor. This multidimensional inversion is efficiently performed by GP in less than 100 compositional simulations. The novelty of this work is to efficiently use the well test data for updating the static models in a fluvial reservoir using a perturbation of the geological models. Moreover, for the first time, a novel optimization method is combined with PPM to consistently update the model parameters with a limited reservoir simulation budget.

Introduction

Transient well testing has traditionally been used to match some simplified mathematical models to the measured pressured data. This provides a good insight into the average properties of the reservoir close to a well (Landa et al., 1996). Nonetheless, the correct well test data interpretation requires the full integration of multi-domain and multi-source information (Kuchuk et al., 2010). However, the underlying assumptions of analytical models, which typically address the statistically homogeneous reservoirs, do not

allow the full translation of the interpreted results between the well testing and other disciplines such as geology and geophysics. Furthermore, the developed analytical models are very limited and are not able to fully describe the complex reservoir heterogeneities. Even if the analytical interpretation gives an average single-point result over heterogeneities, it is crucial to incorporate the dynamic behavior into reservoir characterization. The scope of well testing has evolved from mainly a parameter estimation technique to a more sophisticated discipline for reservoir description (Du and Stewart, 1992). Such advancements clearly require the use of numerical models to simulate transient well test simulations for complex geological scenarios. The term ″geological well testing″ (Corbett et al., 2012; Hamdi, 2012) is referred to the numerical simulations of well tests by setting up the detailed geological models (Massonnat and Bandiziol, 1991; Corbett et al., 1996; Zheng et al., 1996; Corbett et al., 2010). This approach is basically a forward modeling method used in well test designs to understand the impact of the heterogeneity on the well test curves. However, in practice, the existence of the considerable uncertainties in the measurement data, and lack of comprehensive conditioning spatial data urges the development of geology-assisted matching approaches to calibrate the geological model against the well test data (i.e. 'dynamic' data). It is therefore required to efficiently generate and sample the model properties (i.e. facies, porosity, permeability, etc.) using available 'static' and 'dynamic' data. This process requires an assisted history matching process (Denney, 2008). The common methods for changing the reservoir model are usually manual that are endeavoured to achieve by simply multiplying a bulk of cell properties with some constant values. Such approaches are not favourable as they are cumbersome and most importantly the framework of the underlying geology will not be honored. Therefore, this task not only requires (1) an effective numerical optimization framework to automatically and efficiently vary the reservoir parameters, but also requires (2) a geologically consistent process that can preserve the essential geological knowledge of the reservoir model that is captured in the training image.

To the best of our knowledge, the use of well test data in the numerical studies for quantitative integration with reservoir characterization has been limited and qualitative (Landa, 2009). For instance, Hu (2000) used a Gradual Deformation (GD) method to revamp the Gaussian-based reservoir models to match the well test responses. However, GD relies on the Gaussian assumption to combine two Gaussian models and does not always lead to a ″correct″ sampling of the posterior distribution (Caers, 2007). Li et al. (2010) and Han et al. (2014) used the Ensemble Kalman Filter (EnKf) to integrate well test data for the reservoir characterization. However, the linear assumption between measurements and model parameters as it is presumed in EnKf can be problematic. In EnKf the finite ensemble size can create false correlations. A representative spread should be preserved between the ensemble members to avoid excessive variance reduction which this requires a larger ensemble size and a higher computational cost. Additionally, when the Gaussian assumption is violated, poor uncertainty measure is expected (Heidari et al., 2013). Recently, Hamdi et al. (2015a) employed a two-point geostatistical method (Deutsch, 2002) and used the Differential Evolution algorithm (Price et al., 2005) to calibrate a fluvial reservoir model using well test data. However, the use of two-point geostatistics is limiting where the geobody connectivity cannot be preserved in presence of sparse condition hard and soft data.

Multi-Point Statistical (MPS) modeling is an alternative approach to construct the complex geological heterogeneities to better preserve the spatial shape and the connectivity of reservoir facies (Corbett et al., 2012). The MPS can reduce the artifacts and can honour any conditioning data. For building the MPS models the variogram models are replaced by the training images depicting the expected subsurface heterogeneities (Strebelle and Journel, 2001). The training image should be large enough and contains enough replicates that conveys the conditional probabilities. This is to ensure sound geological features with acceptable geometries are reproduced from the MPS simulations. The training image carries the statistical relation of the reservoir facies in more than two points in the reservoir. A training image can be any conceptual model, a seismic-driven map, a digitized satellite image or an outcrop analogue. The training image is scanned by a template or window to store the conditional probabilities. The MPS

approach is based on a sequential algorithm and follows a random path (controlled by a random seed) to assign the reservoir grid facies to create an MPS realization. An example of using MPS in geological well testing is given by Hamdi et al. (2014b) where they used the multi-point geostatistics to update geostatistical realization using a *manual* engineering-based hybridization algorithm. However, this approach is also limiting, as the manual history matching can be cumbersome when we have a large number of facies with complex interactions. Therefore, there is a gap to use multi-point geostatistical techniques in an automated manner to enhance the use of well test data in the reservoir characterization step.

Probability Perturbation Method or PPM (Caers, 2004) is a unique data integration technique that uses the functionality of MPS modeling to perturb the MPS models in a geologically consistent way. PPM is a ″data integration″ method that can integrate prior information with other available data. This method relies on the Journel's tau-model (Journel, 2002) to integrate various data types, each of them being coded into a probability formulation (Caers, 2007). PPM has been successfully used in some uni-dimensional inverse problems (Hoffman and Caers, 2005; Hermans et al., 2013). An alternative to PPM is ″probability conditioning method″ (Jafarpour and Khodabakhshi, 2011) where a soft probability can be used to perturb the realizations. However, the stochastic search algorithm of PPM is probably the best method that can perturb and generate the posterior MPS realizations (Caers et al., 2006; Park et al., 2013) without additional costs that involve in other sampling-based methods such as Metropolis sampling (Mosegaard and Tarantola, 1995). Although PPM has originally been developed for discrete properties, Hu (2008) modified PPM to also include the continuous variables. However, in this paper we only consider PPM to generate and perturb discrete values of the spatial facies realizations.

The use of PPM for quantitative use of well testing in reservoir characterization is treated as an inverse problem. The original PPM includes an outer and an inner loop to search for a realization that can reproduce a dynamic response as the measured data. The outer loop changes the random seed (i.e. creates a new realization), and the inner loop only involves with changing one real variable (r) that controls the perturbation of a reference MPS model. The inner loop is intrinsically a 1D history matching problem which is not computationally expensive to solve. However, when the history matching problem involves with the other parameters such as porosity, permeability, skin factor, etc. the dimensionality of the problem increases which makes the problems more challenging to solve (i.e. the *curse* of dimensionality). For such situations, an efficient optimization algorithm is sought. Two main categories of algorithms exist in the literature to tackle the non-linear history-matching problem. These include gradient-based algorithms (Slater and Durrer, 1971), and population-based approaches (e.g. genetic algorithm (Romero et al., 2000)), differential evolution (Hajizadeh et al., 2009), estimation of distribution algorithms (Abdollahzadeh et al., 2013)). However, the gradient-based algorithms cannot guarantee to find the global solutions and the population-based methods require a large number of simulations. These two shortcomings impose critical limitations as the reservoir models are commonly non-linear and can have many local solutions. Besides, the computational costs of large reservoir models is an obstacle to run a large number of reservoir simulations. Fortunately, there have been some developments in stochastic inversion techniques that are based on some surrogate techniques that only need a limited number of costly function evaluations (simulation runs). Gaussian Process (GP) (Rasmussen and Williams, 2005) is a global optimization technique from this category that has shown great potentials, mainly in high dimensional problems. GP, also known as multi-dimensional kriging, is a black box global optimization technique that is used for the optimization of the costly functions where we can only afford to perform a limited number of function evaluation. The application of GP as a global optimization tool is new to reservoir engineering (Hamdi et al., 2014a; Hamdi et al., 2015b). In the GP approach, an experimental design is used to generate some models from the parameter ranges (training data) to perform well test simulations. A GP surrogate model (or equivalently a multi-dimensional kriging model) is built to represent the well test simulation outputs. The GP is used to propose a sample location (i.e. a set of unknown parameters) that can potentially have the lowest misfit. The proposed sample is passed to the actual simulator. If the acceptable match has not

been acquired, the initial sample pool is augmented by the recent simulated sample and the process repeated until convergence happens or the maximum number of simulations has been reached.

In this paper, it is aimed to calibrate the reservoir models using the pressure buildup well test data by GP inversion for a combination of unknown well/reservoir parameters involving the perturbation parameter r. The seed number will be changed in an outer loop if an acceptable match is not achieved.

Method

PPM

In the PPM method, we are trying to find and combine various probability information using Journel's formulation (Journel, 2002) within the MPS algorithm. As such, if $p(A_m)$ is the marginal or the overall proportion of an event A_m (such as having a facies f_m) that occurs independently of the locations, the MPS algorithm can generate $p(A_m|B)$. This being the probability of having facies f_m in the grid cells by knowing some additional information (i.e. hard data). Moreover, PPM can give $p(A_m|D)$ that is the probability of having facies f_m in the grid cells given the dynamic data. Consequently, the Journel's model is used to provide a straightforward method to combine these probabilities to generate a new MPS realization. In PPM, a new model m_{new} is obtained as $\mathbf{m}_{new} = f(r, \mathbf{m}_0)$ in which $\mathbf{m}_0$ is an initial MPS model that is perturbed using a real value $0<=r<=1$. The coefficient r is used to generate a probability cube that (in combination with the MPS $p(A_m|B)$) creates new soft data for generating a new model $\mathbf{m}_{new}$ i.e. $p(A_m|D) = (1-r)\times i^{(0)}(\mathbf{u}, f_m) + r\times p(A_m)$, in which, subscript $_m$ is an integer represents facies=1, 2,. . .M, A_m denotes $\{i^{(0)}(\mathbf{u}, f_m)=1\}$ and $i^{(0)}(\mathbf{u}, f_m)$ is an indicator model corresponding to the initial realization $\mathbf{m}_0$ which is defined as follows (Hoffman, 2005):

$$i^{(0)}(\mathbf{u}, f_m) = \begin{cases} 1 & \text{if facies } f_m \text{ occurs at location } \mathbf{u} \\ 0 & \text{for all other facies} \end{cases} \quad (1)$$

It should be noted that, if $r=0$, then $p(A_m|D)=0$ or 1, and if $r=1$ then $p(A_m|D) = p(A_m) \in [0,1]$. It is also noted that $\mathbf{m}_0$ by itself is a function of a random seed. This random seed is used to generate an individual equi-probable geostatistical model that itself can be perturbed using PPM and r. Figure 1 shows that with the use of PPM, we can generate multiple reversible realizations with only varying r.

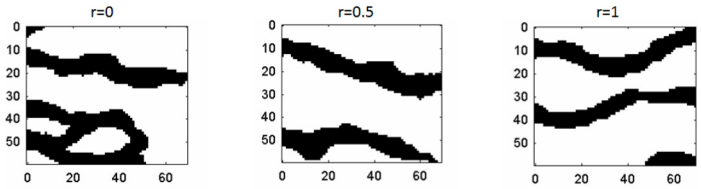


Figure 1—PPM is used to perturb an initial MPS model (r=0) and generate new MPS models (e.g. r=0.5 and r=1).

The value of r therefore, regulates the perturbation from one prior model to another model. Moreover, if variation of r cannot provide the same dynamic response as the measured data, the initial model $\mathbf{m}_0$ is replaced by another equi-probable model by selecting another seed number. Note that although the seed number represents an initial realization of the model, it cannot be used to generate dependent realizations. In other words, the seed number can generate a random walk on the reservoir grid to fill with some discrete facies values, and therefore, the model generated by seed i for instance, is not a perturbation of the model with seed *i-1*.

GP

Optimization using GP modeling is performed with a premise that *a prior* distribution over any finite number of simulations' misfits is multi-Gaussian. Hence, the output or the misfit value ($\mathbf{y}$) can be written as $y=f(\mathbf{x})+\varepsilon$ with $f(\mathbf{x})\sim N(h(\mathbf{x}), \Sigma)$, in which $\mathbf{x}\in\mathbb{R}^d$ is a d-dimensional vector of unknown parameters

(input), N represents a multi-Gaussian distribution with a mean h(**x**) and a covariance matrix Σ. ε is a Gaussian random noise ~N(0, σ^2_n). The function f is usually decomposed into a regression function h(**x**) (that is frequently assumed to be a constant global mean μ) and a simple Gaussian process Z(**x**) (that is a deviation from the global mean μ) with a mean 0, a covariance matrix Σ that has a variance σ^2 i.e. $f(\mathbf{x})=\mu+Z(\mathbf{x})+\varepsilon$. If we ignore the random noise, we can say that for any given **x**, f(**x**) is a sample drawn from $\mu+N(0, \Sigma)$. Additionally, the correlation between Z(**x**) and Z(**x**′) only depends on |**x**-**x**′| that for a popular choice of a squared exponential covariance function can be calculated as (Rasmussen and Williams, 2005):

$$k(\mathbf{x},\mathbf{x}') = \sigma^2 \exp\left(-\sum_{i=1}^{d} \theta_i |x - x'|^2\right) \tag{2}$$

in which, x_i is a simulation model parameter from the sample array **x**, andθ_i is a parameter that shows the importance of x_i on f(**x**).

To build the prior GP model, we need to generate an initial set of parameters $\mathbf{X}=\{\mathbf{x}^1, \mathbf{x}^2, \ldots \mathbf{x}^n\}$ from an experimental design such as Latin Hypercube and perform forward simulations to obtain the corresponding misfit values $\mathbf{Y}=\{y^1, y^2, \ldots, y^n\}$. We use this training dataset to estimate the GP hyperparemters (θ_i, μ and σ^2) by maximizing theLikelihood that $f(\mathbf{x}=\mathbf{x}^j)=y^j$. When the hyperparametrs are obtained the Bayesian inference can help obtain *a posterior* f(**x***)|f(**X**)~N(μ*, σ^{*2}) i.e.

$$\mu^* = \mu + \Sigma(\mathbf{x}^*, \mathbf{X})\Sigma(\mathbf{X}, \mathbf{X})^{-1}(\mathbf{Y} - \mu) \tag{3}$$

$$\sigma^{*2} = \Sigma(\mathbf{x}^*, \mathbf{x}^*) - \Sigma(\mathbf{x}^*, \mathbf{X})\Sigma(\mathbf{X}, \mathbf{X})^{-1}\Sigma(\mathbf{X}, \mathbf{x}^*) \tag{4}$$

The optimization process using GP modeling is to use the current training data set {**X**, **Y**}, fit a prior GP model, and use the Bayesian inference to find an **x*** that can maximize an improvement (I(**x***)) (infill criteria) that we can achieve from the current lowest value of the function (y^{best}). At this stage, no flow simulation is performed, the GP emulator is used to propose this potential point. When this point is found, the actual flow simulation will be performed and the newly simulated point {**x***, y*} will be added to the current training data and the process will be repeated again until a convergence criteria is achieved. The expected improvement function or the infill criterion is defined as (Jones et al., 1998)

$$\mathbf{E}[I(\mathbf{x}^*)] = \left(y^{best} - \mu^*\right)\mathbf{\Phi}\left(\frac{y^{best}-\mu^*}{\sigma^*}\right) + \sigma^*\boldsymbol{\varphi}\left(\frac{y^{best}-\mu^*}{\sigma^*}\right) \tag{5}$$

in which, **E** is the mathematical expected operator, **Φ** is the cumulative density function of standard normal distribution and $\boldsymbol{\varphi}$ is the probability density function of standard normal distribution. The first and the second terms of the above equation promote the *exploitation* and the *exploration* of the GP optimization algorithm respectively. The convergence of the GP algorithm is measured using a regret function that in our case is equal to the current best model with the lowest misfit in each iteration.

Results

Truth case

The example presented in this paper corresponds to the use of well test data in calibrating the geostatistical models in a 2D channelized reservoir model. The training image (TI) is obtained by digitizing the major facies from a satellite image of a section of Magdalena River in Colombia (Figure 2). The training image has 164×203 cells in x and y directions respectively. The training image size is large enough to reproduce the facies shapes and interactions. The training image for this fluvial environment includes four facies ranging from high-quality coarse sand to shale deposits. For this example, it is assumed that the global proportions of the facies are reflected in the facies fractions in the training image.

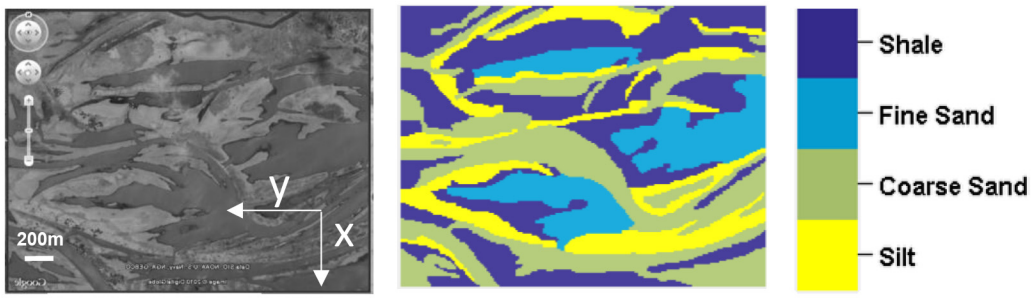


Figure 2—*left*: The satellite image of a small section of Magdalena River in Colombia (Google Erath^{TM}), and *right*: its corresponding 2D training image

The training image (TI) is scanned using a circular template, which can better reproduce the shape even though the training image pattern is anisotropic (Schlumberger, 2012). Although there are some methods that can help find an optimum size of the template (e.g. using the maximum entropy concept as documented by Lyster and Deutsch (2006)), the size is usually obtained by a trial and error approach. In this example, the template size was selected as 50×50. This size was determined after many trials where the continuity of the resulting MPS facies was visually checked using different template radii.

To simulate an MPS model a 2D grid with similar dimensions as the TI (164×203) is assumed. Each cell of the reservoir measures a reservoir volume of 10ft×10ft ×100ft. By taking the facies data for every 40 cells in the TI in both x and y directions as well as the grid cell that contains the subject well (i.e. the middle of the reservoir), a set of hard data is generated. The assumed soft data or the probability map for any facies f_m is obtained by employing a smoothing window on the $\boldsymbol{i}^{(TI)}(\mathbf{u}, f_m)$. Figure 3 shows probability maps for various facies that are used within the MPS algorithm.

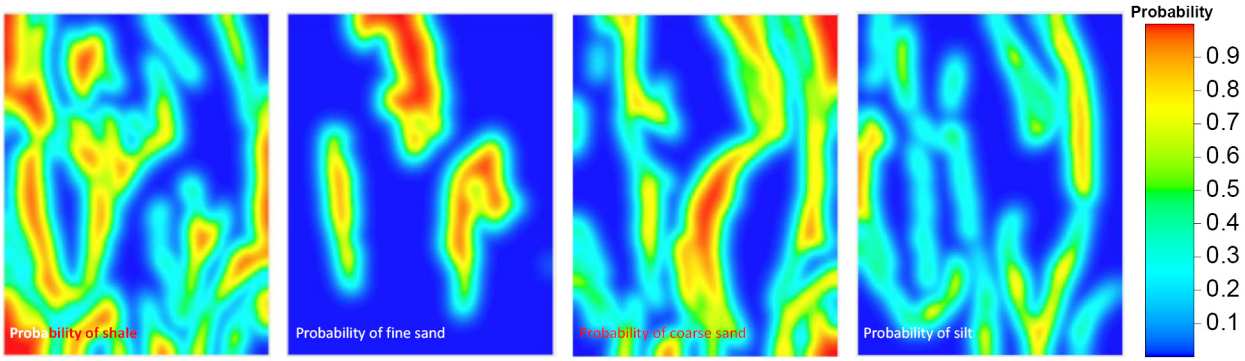


Figure 3—The probability maps for various facies (soft data).

The soft and hard data sets are used to generate a single MPS model using an individual seed number of 211171. The resulting model is perturbed using PPM with r=0.63. The well test response of a fully penetrated vertical well that is placed in the middle of the model is simulated using a commercial reservoir simulator (tNavigator). The well is put under a constant surface production rate of 1000 Mscf/day in an under-saturated gas condensate fluid system lasting for 8 days that is then followed with a buildup period of 24 days. The initial reservoir pressure is 5500 psia that is well above the dew point pressure (3733 psi) and is maintained during the well test simulations. This model serves as a real reservoir (or a truth case) and its corresponding well test response is considered as the real measured well test data. The truth model and its seed number are blinded and we try to find a model that can reproduce a well test response similar to the truth model. The real reservoir parameters for the truth model are listed in Table 1. The porosity values of the facies are kept unchanged, i.e. φ_{shale}=0.01, φ_{silt}=0.05, $\varphi_{corase\ sand}$=0.2 and $\varphi_{fine\ sand}$=0.1.

Table 1—The truth model and parameter ranges used in this study.

Parameters	Ranges	Truth case
K_{shale}	0.001 (fixed)	0.001
X1: K_{silt}	[0.001,0.1]	0.01
X2: $K_{coarse\ sand}$	[10-1000]	100
X3: $K_{fine\ sand}$	[1-100]	10
X4: Non-Darcy D-factor	[1E-5,0.1]	5E-3 day/Mscf
X5: *r*	[0,1]	0.63

Figure 4 shows the facies model (Figure 4: left) of the truth case and its corresponding well test response (Figure 4: right). The well test derivative response shows an apparent radial flow regime with a zero slope line that is followed by a rather long ¼ slope trend line and a final ½ slope trend line. The radial part represents a permeability of around 8 md and a skin factor of 3.5. Both of these values do not correspond well to the values of the skins and the facies permeability used in the truth model. The main reason for this is that the well is located next to a low permeable boundary and the well test response does not record the early times (<0.01 hr) and therefore, the values do not correspond completely to the actual skin and the permeability value. This example shows that the analytical well test interpretations should be used with great care to include the interpretation uncertainty for the reservoir engineering applications that require rather accurate reservoir parameters' estimations. The ¼ and ½ slope trend lines are also difficult to relate with a representative analytical model. Therefore, the purpose of this study is to promote the use of numerical interpretation to characterize a reservoir model that honors the measured static and dynamic data.

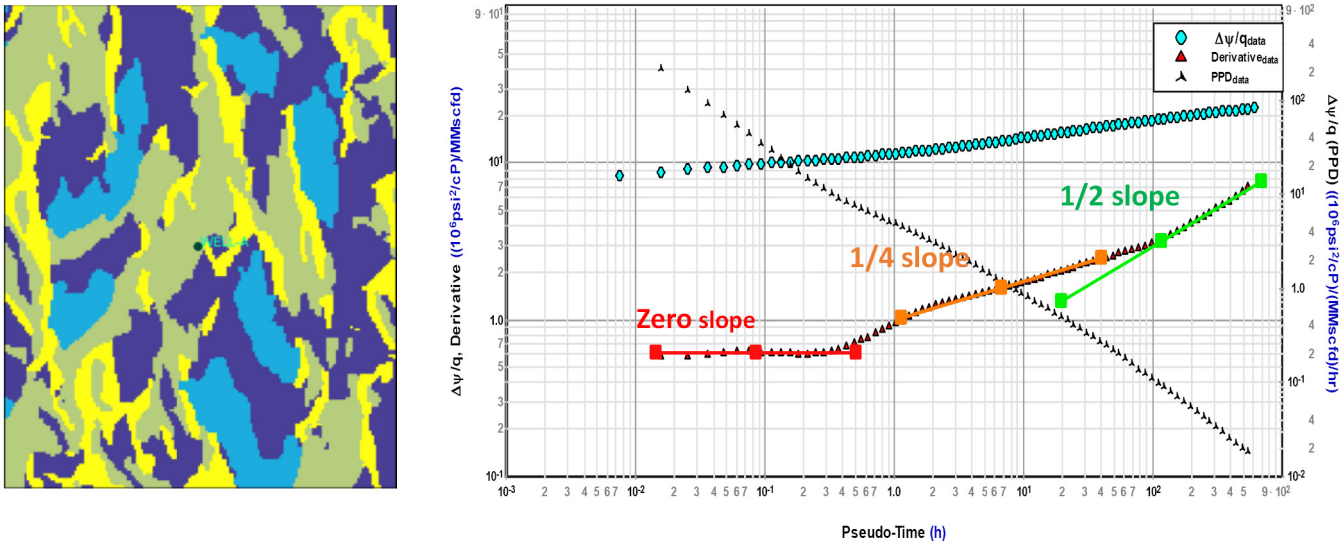


Figure 4—The truth case facies model (left) and its corresponding buildup pressure response (right).

Figure 5 shows the streamlines in different timesteps during the production in the truth model. The figure displays that after 0.1 hr the diffusion front created by the subject well has reached a tight boundary in its right (Figure 5: left). In the later times, the front will propagate in the reservoir with the preference to diffuse in the high permeable rocks (Figure 5: middle and right). The anisotropic growth of the diffusion front in this case can highlight the critical assumption of the analytical methods where a homogenous model with isotropic circular expansion of the fronts are presumed in almost all calculations.

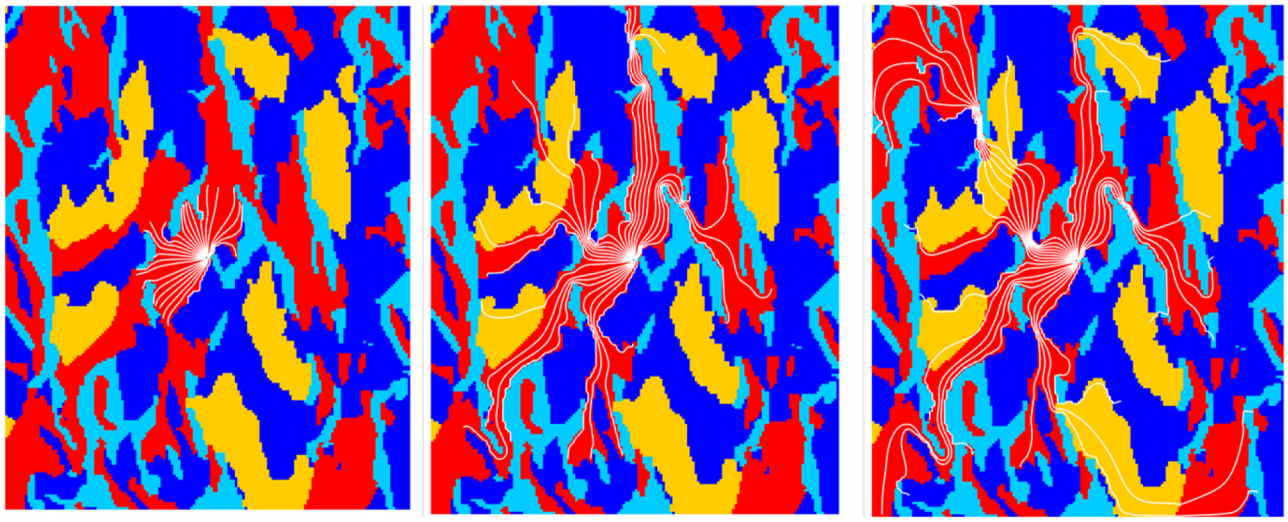


Figure 5—The propagation of the diffusion front within the truth model at different drawdown times: 0.1 hr (left), 4 hr (middle) and 8 days(right). The streamlines follow the high permeable facies packages.

Inversion

The GP modeling approach is coupled with the PPM to set an inversion framework for the reservoir description using well test data. In this example, we generate an initial population of 20 models sampled from the parameter space using Latin Hypercube experimental design. The GP is set to run for 4 trials (each with different initial populations and seed numbers), where each trial is also limited to 100 reservoir simulations. The misfit function (M) is defined over buildup timesteps as follows:

$$M = \sum \left(\frac{\log(\Delta p^{truth}) - \log(\Delta p^{sim})}{\log(\Delta p^{truth})} \right)^2 + \sum \left(\frac{\log\left(\Delta p_{der}^{truth}\right) - \log(\Delta p_{der}^{sim})}{\log(\Delta p_{der}^{truth})} \right)^2 \quad (6)$$

where, $\Delta p = p_{ws} - p_{wf@tp}$, t_p is the production time prior to buildup, p_{ws} is the shut-in pressure, $p_{wf@tp}$ is the wellbore pressure at the shut-in time, the bar sign represents the arithmetic mean, Δp_{der} is the Bourdet derivative (Bourdet, 2002), and the superscripts ^{truth} and ^{sim} correspond to the well test responses of the truth and the simulated models respectively. The log-log plot of Δp and Δp_{der} vs time is sensitive to the permeability and the lateral variations of the facies, and the Cartesian plot indicates overall quality of the match and is sensitive to the reservoir volume. Although for general cases, the historical data found in the Cartesian domain can be added to the misfit function definition, in this example only the buildup data is used as a measure of convergence. The historical data is checked afterwards to accept or reject the acquired match.

The compositional well test simulations are performed using tNavigator (Rock Flow Dynamic, 2016), which employs accelerated algorithms on many cores using shared and distributed memory computing that can improve the simulations' accuracy and the costs (Christie et al., 2013).Figure 6 shows the results of the best models (lowest misfits) in different GP trials. Obviously, the facies model and the estimated reservoir parameters are different from the truth case model and its well test response. This clearly shows the non-unique nature of the problem where multiple combinations of the reservoir parameters and the facies distributions can produce the same well test output. However, using a consentient geoengineering approach we aim to integrate all available data within a unified framework for a better reservoir description with a potential reduction of the modeling uncertainty. The implication of having multiple models with similar output, is on the future production and the development scenarios where different models could imply different production strategies and future EOR processes.

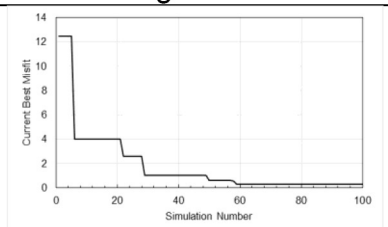
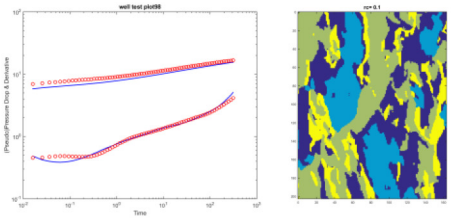
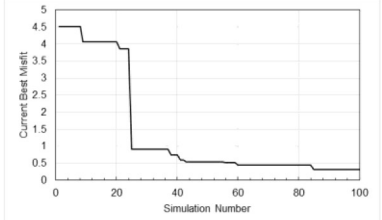
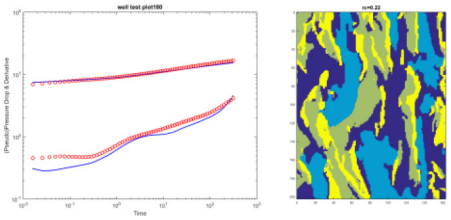
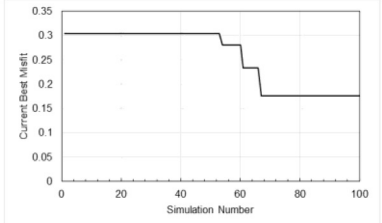
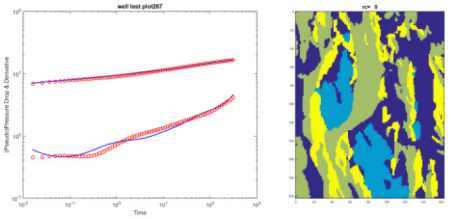
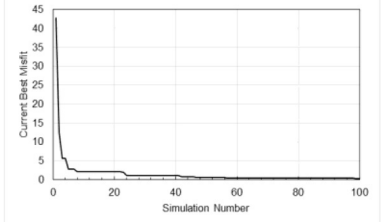
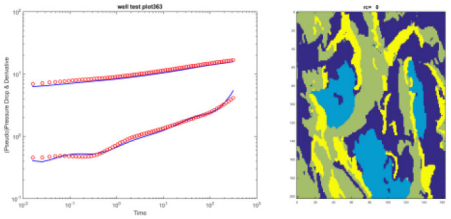
	GP convergence	Match quality	Best fit
Trial 1			X1=0.029624 X2=71.26944 X3=27.946813 X4=0.000152 X5=0.101475555 Seed=10748845
Trial 2			X1=0.001 X2=112.448664 X3=6.667296 X4=0.007813 X5=0.216507923 Seed=6758207
Trial 3			X1=0.001 X2=81.23118 X3=1 X4=0.000658 X5=0 Seed=908761
Trial 4			X1=0.1 X2=91.331653 X3=5.933597 X4=0.002941 X5=0 Seed=10431847

Figure 6—The convergence performance of the GP, and corresponding best cases from 4 different trials. Each trial represents a different seed number.

Although all best models obtained from different trials show relatively good match qualities on the log-log plots (Figure 6), only the cases in trial 1 and 3 (approximately) can be accepted because the match quality in the corresponding Cartesian pressure plots should also be considered (Figure 7). For the models in trial cases 2 and 4, there are larger pressure differences between the initial pressure at the start of the simulations and the buildup stabilization pressures, which can indicate the lack of supporting connected volumes in those models. The issue of having a Cartesian pressure history mismatch would probably improve if we could include the Cartesian pressure data in the misfit formulae.

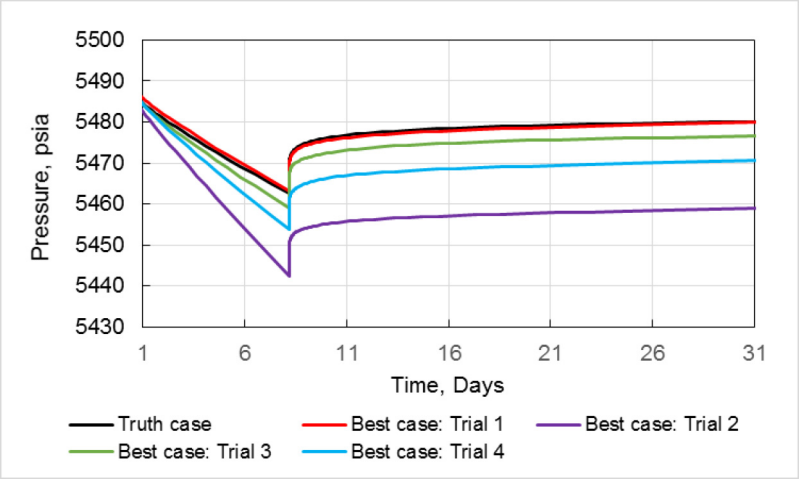


Figure 7—The pressure history plots for best case candidates obtained from different GP trials as indicated in Figure 6.

The optimization of a multi-objective problem using a weighted approach (equation 6) could be problematic particularly when the objectives are conflicting. For these situations, a multi-objective optimization algorithm is preferred (Deb et al., 2000; Beume et al., 2007). This is an interesting matter for a future study, where we aim to test our algorithm to include the pressure history data for multiple well tests using a multi-objective optimization algorithm.

Conclusions

In this paper, the PPM was used to calibrate multi-point geostatistical models using pressure transient data. The PPM could consistently perturb the geostatistical models to generate the new models that honor the trend and the statistical properties of the underlying training images. On the other hand, the GP was employed for the inversion process, where only limited well test simulations were required. The algorithm was successfully tested on a 2D channelized reservoir model with 5 unknown parameters. The GP provided some solutions that could honor the conditioning hard and soft data as well as the truth well test responses. The difference observed between the resulting models and the truth parameters could stress the non-unique nature of the inverse problems where we should expect multiple solutions to reproduce the same outputs. The results also confirmed that although a good match might be obtained using the log-log plots, the pressure history data should also be included in the misfit function to contain the impact of the reservoir volume for more robust optimization.

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